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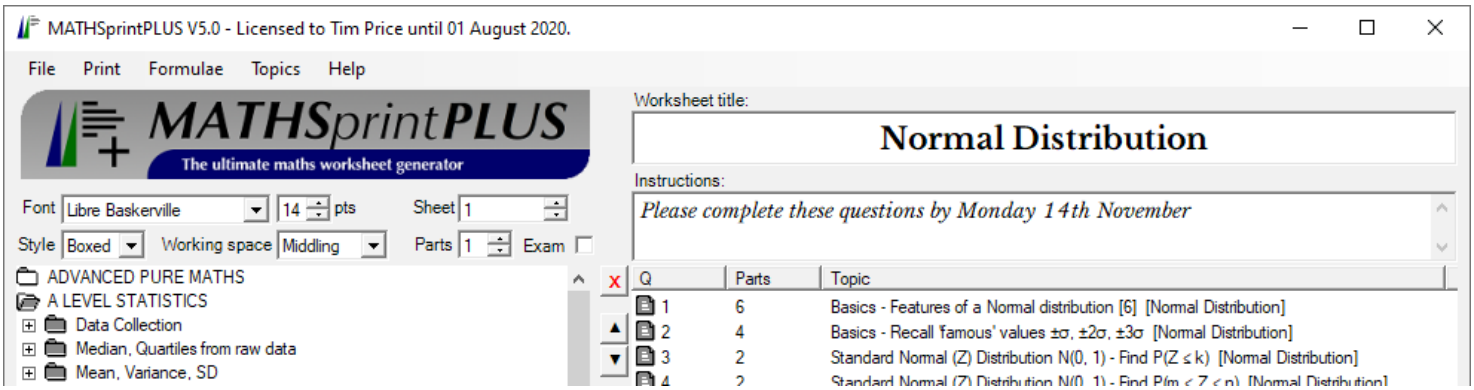
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File
Print
Formulae
Topics
Help
MATHSprintPLUS
The ultimate maths worksheet generator
Worksheet title:
Normal Distribution
Instructions:
Please complete these questions by Monday 14th November
Font
Libre Baskerville
14
pts
Sheet
1
Style
Boxed
Working space
Middling
Parts
1
Exam
ADVANCED PURE MATHS
A LEVEL STATISTICS
Data Collection
Median, Quartiles from raw data
Mean, Variance, SD
Q
Parts
Topic
1
6
Basics - Features of a Normal distribution [6] [Normal Distribution]
2
4
Basics - Recall 'famous' values ±σ, ±2σ, ±3σ [Normal Distribution]
3
2
Standard Normal (Z) Distribution N(0, 1) - Find P(Z ≤ k) [Normal Distribution]

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*1/(√5 ± 3√3), etc.
*Misc 1
*(2 ± √5)/(3 ± √5), etc.
*(2 ± 4√5)/(3 ± 2√5), etc.
*(√5 + √3)/(√5 - √3), etc.
*(√5 + 3√3)/(√5 - 3√3), etc.
*Misc 2

PURE: QUADRATICS

Solving quadratic equations

Factorise and solve
*(x + 2)(x + 3) = 0, etc.
*x(x + 4) = 0, etc.
*x^2 + 4x = 0, etc.
*x^2 ± 5x ± 6 = 0, etc.
*x^2 + 10x + 25 = 0, etc.
*x^2 - 25 = 0, etc.
*Misc x^2 ± Ax ± B = 0
*$3x^2$ - 7x + 2 = 0, etc.

Rearrange, factorise and solve
*(x - 1)(x + 2) = 4
*x + 6/x = 5, etc.
*x - 4/(x + 2) = 1, etc.
*3/(x + 7) - 3/(x + 9) = 2, etc.
*Misc. equations

Solve using the formula
*x^2+x-5=0, etc., surd form
*$2x^2$+x-5=0, etc., surd form
*x^2+x-5=0, etc., 3 sig figs
*$2x^2$+x-5=0, etc., 3 sig figs

Complete the square and solve
*$(x + p)^2 + q$
*$a(x + p)^2 + q$

Functions

Evaluate f(a)
*1 step
*2 steps with x^2
*2 steps with √x
*2 steps with x^2, √x
*3 steps with x^2
*3 steps with √x
*3 steps with x^2, √x
*Misc 2 and 3 step function
*Rational function 1
*Rational function 2
*Misc rational function
*Misc function

Solve f(x) = a
*1 step
*2 steps with x^2
*2 steps with √x
*2 steps with x^2, √x
*3 steps with x^2
*3 steps with √x
*3 steps with x^2, √x
*Misc 2 and 3 step function
*Rational function 1
*Rational function 2
*Misc rational function
*Misc function

Solve f(x) = 0
*(x + 3)(x - 4) = 0
*x^2 - x - 12 = 0
*x(x + 3)(x - 4) = 0
*x^3 - x^2 - 12x = 0
*(x + 1)(x + 3)(x - 4) = 0
*Misc polynomial factorised
*Misc polynomial unfactorised
*Misc polynomial

Solve f(x) = g(x)
*x^2 + 2x = 3x + 12
*x^2 + 2x - 5 = 3x + 7
*Misc quadratic
*x^3 - 12x = x^2
*x^3 + $2x^2$ - 10x = $3x^2$ + 2x
*Misc cubic
*Misc quadratic/cubic

Solve quadratics with substitution
*$u = x^2$
*$u = x^3$
*u = √x
*$u = \sqrt[3]{x}$
*$u = k^x$
*Misc substitution

Quadratic graphs

Complete the square to:
*Find line of symmetry $(x+p)^2+q$
*Find line of symmetry $a(x+p)^2+q$
*Find min/max value $(x+p)^2+q$
*Find min/max value $a(x+p)^2+q$
*Find co-ordinates of vertex $(x+p)^2+q$
*Find co-ordinates of vertex $a(x+p)^2+q$
*Find a misc feature $(x+p)^2+q$
*Find a misc feature $a(x+p)^2+q$
*List co-ordinates of vertex and all axis intersections $(x+p)^2+q$
*List co-ordinates of vertex and all axis intersections $a(x+p)^2+q$

Find the quadratic equation
*Given the vertex and y-intercept $(x+p)^2+q$
*Given the vertex and y-intercept $a(x+p)^2+q$
*Given x- and y-axis intersections $(x+p)^2+q$
*Given x- and y-axis intersections $a(x+p)^2+q$
*From a sketch graph $y = (x+p)^2+q$
*From a sketch graph $y = a(x+p)^2+q$

*From a sketch graph $y = x^2 + bx + c$
*From a sketch graph $y = ax^2 + bx + c$
Sketch the quadratic graph
*From the equation $y = (x+p)^2+q$
*From the equation $y = a(x+p)^2+q$
*From the equation $y = x^2 + bx + c$
*From the equation $y = ax^2 + bx + c$

The discriminant

Discriminant basics
*Calculate the discriminant
*Find the number of roots of a quadratic
Discriminant equations and inequalities
*Find k in $ax^2 + bx + k = 0$ given the number of real roots
*Find k in $ax^2 + kx + c = 0$ given the number of real roots

PURE: EQUATIONS and INEQUALITIES

Linear simultaneous equations

Linear sim eqn, integer, no multiply
*2x+3y=7, 4x+3y=11 (+ve x,y)
*2x+3y=7, 4x-3y=11 (+ve x,y)
*2x+3y=7, 4x±3y=11 (+ve x,y)
*2x+3y=7, 4x+3y=11 (±ve x,y)
*2x+3y=7, 4x-3y=11 (±ve x,y)
*2x+3y=7, 4x±3y=11 (±ve x,y)
Linear sim eqn, integer, multiply one equation ×2 or ×3
*2x+y=5, 3x+2y=8 (+ve x,y)
*2x+y=5, 3x-2y=4 (+ve x,y)
*2x+y=5, 3x±2y=4 (+ve x,y)
*2x+y=5, 3x+2y=8 (±ve x,y)
*2x+y=5, 3x-2y=4 (±ve x,y)
*2x+y=5, 3x±2y=4 (±ve x,y)
Linear sim eqn, integer, multiply one equation ×4 or more
*2x+y=5, 3x+4y=10 (+ve x,y)
*2x+y=5, 3x-4y=2 (+ve x,y)
*2x+y=5, 3x±4y=2 (+ve x,y)
*2x+y=5, 3x+4y=10 (±ve x,y)
*2x+y=5, 3x-4y=2 (±ve x,y)
*2x+y=5, 3x±4y=2 (±ve x,y)
Linear sim eqn, integer, multiply both equations
*2x+3y=7, 3x+4y=10 (+ve x,y)
*2x+3y=7, 3x-4y=2 (+ve x,y)
*2x+3y=7, 3x±4y=2 (+ve x,y)
*2x+3y=7, 3x+4y=10 (±ve x,y)
*2x+3y=7, 3x-4y=2 (±ve x,y)
*2x+3y=7, 3x±4y=2 (±ve x,y)
Linear sim eqn, (half)-integer, no multiply
*2x+3y=7, 4x+3y=11 (+ve x,y)
*2x+3y=7, 4x-3y=11 (+ve x,y)
*2x+3y=7, 4x±3y=11 (+ve x,y)
*2x+3y=7, 4x+3y=11 (±ve x,y)
*2x+3y=7, 4x-3y=11 (±ve x,y)
*2x+3y=7, 4x±3y=11 (±ve x,y)
Linear sim eqn, (half)-integer, multiply one equation ×2 or ×3
*2x+y=5, 3x+2y=8 (+ve x,y)
*2x+y=5, 3x-2y=4 (+ve x,y)
*2x+y=5, 3x±2y=4 (+ve x,y)
*2x+y=5, 3x+2y=8 (±ve x,y)
*2x+y=5, 3x-2y=4 (±ve x,y)
*2x+y=5, 3x±2y=4 (±ve x,y)
Linear sim eqn, (half)-integer, multiply one equation ×4 or more
*2x+y=5, 3x+4y=10 (+ve x,y)
*2x+y=5, 3x-4y=2 (+ve x,y)
*2x+y=5, 3x±4y=2 (+ve x,y)
*2x+y=5, 3x+4y=10 (±ve x,y)
*2x+y=5, 3x-4y=2 (±ve x,y)
*2x+y=5, 3x±4y=2 (±ve x,y)
Linear sim eqn, (half)-integer, multiply both equations
*2x+3y=7, 3x+4y=10 (+ve x,y)
*2x+3y=7, 3x-4y=2 (+ve x,y)
*2x+3y=7, 3x±4y=2 (+ve x,y)
*2x+3y=7, 3x+4y=10 (±ve x,y)
*2x+3y=7, 3x-4y=2 (±ve x,y)
*2x+3y=7, 3x±4y=2 (±ve x,y)

Quadratic simultaneous equations

Elimination, rational solutions
*$y=x^2$, y=2x+3, etc.
*$y=x^2+7x+9$, y=2x+3, etc.
Easy substitution, rational solutions
*$xy = k$, $x + ny = p$
*$ax^2 + by^2 = k$, $x + ny = p$
*$ax^2 - by^2 = k$, $x + ny = p$
*$ax^2 \pm bxy = k$, $x + ny = p$
*$ax^2 \pm bxy + cy^2 = k$, $x + ny = p$
*$ax^2 \pm bxy - cy^2 = k$, $x + ny = p$
*$xy + dx + ey = k$, $x + ny = p$
*$ax^2 + by^2 + dx + ey = k$, $x + ny = p$
*$ax^2 - by^2 + dx + ey = k$, $x + ny = p$
*$ax^2 \pm bxy + dx + ey = k$, $x + ny = p$
*$ax^2 \pm bxy + cy^2 + dx + ey = k$, $x + ny = p$
*$ax^2 \pm bxy - cy^2 + dx + ey = k$, $x + ny = p$
Harder substitution, rational solutions
*$xy = k$, $mx + ny = p$
*$ax^2 + by^2 = k$, $mx + ny = p$
*$ax^2 - by^2 = k$, $mx + ny = p$
*$ax^2 \pm bxy = k$, $mx + ny = p$
*$ax^2 \pm bxy + cy^2 = k$, $mx + ny = p$

*$ax^2 \pm bxy - cy^2 = k$, $mx + ny = p$
*$xy + dx + ey = k$, $mx + ny = p$
*$ax^2 + by^2 + dx + ey = k$, $mx + ny = p$
*$ax^2 - by^2 + dx + ey = k$, $mx + ny = p$
*$ax^2 \pm bxy + dx + ey = k$, $mx + ny = p$
*$ax^2 \pm bxy + cy^2 + dx + ey = k$, $mx + ny = p$
*$ax^2 \pm bxy - cy^2 + dx + ey = k$, $mx + ny = p$

Linear inequalities

Solving linear inequalities (±ve x coeff)
*$\pm x \pm b < c$, $\pm x \pm b > c$
*$\pm ax < c$, $\pm ax > c$
*$\pm ax \pm b < c$, $\pm ax \pm b > c$
*$\pm a(x \pm b) < c$, $\pm a(x \pm b) > c$
*Misc

Solving linear inequalities and plotting (±ve x coeff)
*$\pm x \pm b < c$, $\pm x \pm b > c$
*$\pm ax < c$, $\pm ax > c$
*$\pm ax \pm b < c$, $\pm ax \pm b > c$
*$\pm a(x \pm b) < c$, $\pm a(x \pm b) > c$
*Misc

Quadratic inequalities

Simple x^2
*eg $x^2 > 4$
*eg $2x^2+4 < 22$

Factorised
*$(x - 4)(x + 5) > 0$, etc.
*$-(x - 4)(x + 5) > 0$, etc.
*$\pm(x - 4)(x + 5) > 0$, etc.

Factorisable
*$x^2 + 5x - 6 > 0$, etc.
*$-x^2 - 5x + 6 > 0$, etc.
*$\pm x^2 \pm 5x \pm 6 > 0$, etc.
*$x^2 - 6 > -5x$, $x(x - 5) > -6$, etc.

Multiply through by x^2
*$5 > 20/x$, $1 + 6/x^2 < -5/x$, etc.

PURE: GRAPHS and TRANSFORMATIONS

Graph sketching - basic shapes

Powers of x
*$y=x$, x^2, x^3 [3]
*$y=1/x$, $1/x^2$ [2]
*$y=x$, x^2, x^3, $1/x$, $1/x^2$ [5]
*$y=\sqrt{x}$, $\sqrt[3]{x}$ [2]
*Misc [7]

Graph sketching - cubics

Sketch a cubic
*Sketch $y = (x + 3)(x - 2)(x - 4)$, etc.
*Sketch $y = -(x + 3)(x - 2)(x - 4)$, etc.
*Sketch $y = (x + 3)^2(x - 2)$, etc.
*Sketch $y = -(x + 3)^2(x - 2)$, etc.
*Sketch $y = (x + 3)^3$, etc.
*Sketch $y = -(x + 3)^3$, etc.
*Sketch misc factorised
*Factorise and sketch +ve cubic with 3 distinct roots
*Factorise and sketch -ve cubic with 3 distinct roots
*Factorise and sketch +ve cubic with double repeated root
*Factorise and sketch -ve cubic with double repeated root

Suggest an equation for a cubic
*$y = (x + 3)(x - 2)(x - 4)$, etc.
*$y = -(x + 3)(x - 2)(x - 4)$, etc.
*$y = (x + 3)^2(x - 2)$, etc.
*$y = -(x + 3)^2(x - 2)$, etc.
*$y = (x + 3)^3$, etc.
*$y = -(x + 3)^3$, etc.
*Misc cubic

Graph sketching - quartics

Sketch a quartic
*Sketch $y = (x + 3)(x - 2)(x - 4)(x + 1)$, etc.
*Sketch $y = -(x + 3)(x - 2)(x - 4)(x + 1)$, etc.
*Sketch $y = (x + 3)^2(x - 2)(x - 4)$, etc.
*Sketch $y = -(x + 3)^2(x - 2)(x - 4)$, etc.
*Sketch $y = (x + 3)^3(x - 2)$, etc.
*Sketch $y = -(x + 3)^3(x - 2)$, etc.
*Sketch misc factorised
*Factorise and sketch +ve quartic with four distinct roots
*Factorise and sketch -ve quartic with four distinct roots
*Factorise and sketch +ve quartic with double repeated root
*Factorise and sketch -ve quartic with double repeated root

Suggest an equation for a quartic
*$y = (x + 3)(x - 2)(x - 4)(x + 1)$, etc.
*$y = -(x + 3)(x - 2)(x - 4)(x + 1)$, etc.
*$y = (x + 3)^2(x - 2)(x - 4)$, etc.
*$y = -(x + 3)^2(x - 2)(x - 4)$, etc.
*$y = (x + 3)^3(x - 2)$, etc.
*$y = -(x + 3)^3(x - 2)$, etc.
*Misc quartic

Points of intersection

Number of intersections by sketching
*Linear and quadratic
*Linear and cubic
*Linear and $y=1/x$
*Linear and $y=1/x^2$
*Linear and misc.
*Quadratic and cubic
*Quadratic and $y=1/x$

*Quadratic and $y=1/x^2$
*Cubic and $y=1/x$
*Cubic and $y=1/x^2$
*Quadratic/cubic and misc.

Transforming graphs

Basics
*Describe single-step transformation of $y = f(x)$
*Apply single-step transformation to $y = f(x)$
*Describe two-step transformation of $y = f(x)$
*Apply two-step transformation to $y = f(x)$

Transform a function algebraically (1 step)
*Translate: given $f(x) = x^n$, find $f(x \pm k)$
*Translate: given $f(x) = x^n$, sketch $f(x \pm k)$
*Translate $f(x) = x^n$ given a translation vector
*Translate: transform a point on $y = f(x)$ to $y = f(x \pm k)$
*Stretch: given $f(x) = x^n$, find $kf(x)$ or $f(kx)$
*Stretch: given $f(x) = x^n$, sketch $kf(x)$ or $f(kx)$
*Stretch $f(x) = x^n$ given a scale factor and direction
*Stretch: transform a point on $y = f(x)$ to $y = kf(x)$ or $f(kx)$
*Reflect: given $f(x) = x^n$, find $-f(x)$ or $f(-x)$
*Reflect: given $f(x) = x^n$, sketch $-f(x)$ or $f(-x)$
*Reflect $f(x) = x^n$ given a reflection line
*Reflect: transform a point on $y = f(x)$ to $y = -f(x)$ or $f(-x)$
*Misc: given $f(x) = x^n$, find $f(x \pm k)$, $\pm kf(x)$ or $f(\pm kx)$
*Misc: given $f(x) = x^n$, sketch $f(x \pm k)$, $\pm kf(x)$ or $f(\pm kx)$
*Misc algebraic transformation of $f(x) = x^n$
*Misc: transform a point on $y = f(x)$

PURE: STRAIGHT-LINE GRAPHS

Line segments

Line segments in positive quadrant
*Gradient
*Midpoint
*Length
*Gradient and mid-point
*Gradient, mid-point & length

Line segments in any quadrant
*Gradient
*Midpoint
*Length
*Gradient and mid-point
*Gradient, mid-point & length

Features of linear graphs

Gradient and y-intercept from $y = mx + c$
*Identify integer gradient
*Identify integer y-intercept
*Identify integer gradient and y-intercept
*Identify fractional gradient
*Identify fractional y-intercept
*Identify fractional gradient and y-intercept

Find x-intercept from $y = mx + c$
*Identify integer x-intercept
*Identify fractional x-intercept

Converting between $y = mx + c$ and $ax + by + c = 0$
*Rewrite integer $y=mx + c$ as $ax + by + c = 0$
*Rewrite fractional $y=mx + c$ as $ax + by + c = 0$
*Rewrite $ax + by + c = 0$ as integer $y=mx + c$
*Rewrite $ax + by + c = 0$ as fractional $y=mx + c$

Gradient and y-intercept from $ax + by + c = 0$
*Identify integer gradient
*Identify integer y-intercept
*Identify integer gradient and y-intercept
*Identify fractional gradient
*Identify fractional y-intercept
*Identify fractional gradient and y-intercept

Find x-intercept from $ax + by + c = 0$
*Identify integer x-intercept
*Identify fractional x-intercept

Find the equation of a linear graph

Find equation given gradient m and one point
*Integer m; $y = mx + c$
*Integer m; $ax + by + c = 0$
*Fractional m; $y = mx + c$
*Fractional m; $ax + by + c = 0$

Find equation given two points
*Integer m; $y = mx + c$
*Integer m; $ax + by + c = 0$
*Fractional m; $y = mx + c$
*Fractional m; $ax + by + c = 0$

Parallel and perpendicular lines

Test if lines are parallel, perpendicular or neither
*Parallel or not: $y = mx + c$, $y = nx + d$
*Parallel or not: $ax + by + c = 0$, $dx + ey + f = 0$
*Parallel or not: $y = mx + c$, $ax + by + c = 0$
*Parallel or not: misc equations
*Perpendicular or not: $y = mx + c$, $y = nx + d$
*Perpendicular or not: $ax + by + c = 0$, $dx + ey + f = 0$
*Perpendicular or not: $y = mx + c$, $ax + by + c = 0$
*Perpendicular or not: misc equations
*Parallel, perpendicular or neither: $y = mx + c$, $y = nx + d$
*Parallel, perpendicular or neither: $ax + by + c = 0$, $dx + ey + f = 0$
*Parallel, perpendicular or neither: $y = mx + c$, $ax + by + c = 0$
*Parallel, perpendicular or neither: misc equations

Give the equation of a parallel or perpendicular line through a point
*Parallel line as y = mx + c
*Parallel line as ax + by + c = 0
*Perpendicular line as y = mx + c
*Perpendicular line as ax + by + c = 0
*Parallel/perpendicular line as y = mx + c
*Parallel/perpendicular line as ax + by + c = 0
Find the perpendicular bisector of a line segment
*Perpendicular bisector as y = mx + c
*Perpendicular bisector as ax + by + c = 0

PURE: CIRCLES

Midpoints and perpendicular bisectors
Find the midpoint of a line segment
*Midpoint of points in +ve quadrant
*Midpoint of points in any quadrant
Find the perpendicular bisector of a line segment
*Perpendicular bisector as y = mx + c
*Perpendicular bisector as ax + by + c = 0

Circle equation
Give the equation of a circle
*Centre O, radius r
*Centre (a, b), radius r
*Centre O passing through given point
*Centre (a, b) passing through given point
*With given diameter
Give information about a circle
*Radius of a circle centre O
*Radius (integer) and centre of a circle
*Radius (surd) and centre of a circle
*Radius (integer) and centre by completing the square
*Radius (surd) and centre by completing the square

Circle sketch graphs
Sketch a circle from its equation
*Completed square form
*By completing the square
Give the equation of a circle from a sketch
*Completed square form
*By completing the square

Equation of radius and tangent
Radius of a circle
*Gradient of radius, centre O
*Equation of radius, centre O
*Gradient of radius, centre (a, b)
*Equation of radius, centre (a, b)
Tangent to a circle
*Gradient of tangent, centre O
*Equation of tangent, centre O
*Gradient of tangent, centre (a, b)
*Equation of tangent, centre (a, b)

Intersections of straight lines and circles
Solve intersection of circle with x=k, y=k
*Circle and x- or y-axis
*Circle and y=k, x=k
Number of intersections of circle and line
*Circle and y = mx + c
*Circle and ax + by + c = 0
Solve intersection of circle with line
*Circle and y = mx + c, 2 solutions
*Circle and ax + by + c = 0, 2 solutions
*Circle and y = mx + c, 1 solution
*Circle and ax + by + c = 0, 1 solution
*Circle and y = mx + c, 1 or 2 solutions
*Circle and ax + by + c = 0, 1 or 2 solutions
Solve for K: line intersects circle 0/1/2 times
*$ax + by + c = 0$ and $(x-p)^2 + (y-q)^2 = K$
*$ax + by + c = 0$ and $(x-K)^2 + (y-q)^2 = r^2$
*$ax + by + c = 0$ and $(x-p)^2 + (y-K)^2 = r^2$
*$ax + by + K = 0$ and $(x-p)^2 + (y-q)^2 = r^2$
*$y = Kx$ and $(x-p)^2 + (y-q)^2 = r^2$
*Misc

PURE: ALGEBRAIC METHODS

Algebraic fractions
Numeric common factor
*2x/6, etc.
*2(x+2)/6, etc.
*(2x+4)/6, etc.
*(2x+4)/6x, etc.
*(2x+4)/(6x-2), etc.
*Misc
Algebraic common factor
*x(x+2)/6x, etc.
*$(x^2+2x)/6x$, etc.
*x(x+2)/x(x-3), etc.
*$(x^2+2x)/(x^2-3x)$, etc.
*$(2x^2+4x)/(4x^2-12x)$, etc.
*Misc 1
*(x+2)(x+1)/(x+2), etc.
*$(x^2+3x+2)/(x+2)$, etc.
*$(x^2-4)/(x\pm2)$, etc.
*$(x^2-4)/(x^2\pm2x)$, etc.
*(x+2)(x+1)/(x+3)(x+2), etc.
*$(x^2+3x+2)/(x^2+5x+6)$, etc.
*Misc 2
*$(2x^2-5x-3)/(2x-3)(x-3)$, etc.
*$(2x^2-5x-3)/(2x^2-9x+9)$, etc.

Polynomial arithmetic
Polynomial basics

*Order
*Add cubics
*Subtract cubics
Multiply polynomials
*Multiply linear by quadratic
*Multiply linear by cubic
*Multiply quadratic by quadratic
*Multiply quadratic by cubic
*Multiply misc.
Divide polynomials
*Divide quadratic by x + 3, etc.
*Divide quadratic by 3x - 2, etc.
*Divide cubic by x + 3, etc.
*Divide cubic by 3x - 2, etc.
*Divide quartic by x + 3, etc.
*Divide quartic by 3x - 2, etc.
*Divide quintic by x + 3, etc.
*Divide quintic by 3x - 2, etc.

Factor and remainder theorems
Factor theorem
*Is linear a factor of cubic?
*Factorise cubic into linears
*Solve for one unknown
*Solve for two unknowns
Remainder theorem
*Remainder when cubic is divided by linear

Mathematical proof
Algebra toolkit for proof
*Specify even, odd numbers, multiples, etc. [10]
*Specify consecutives, even/odd squares, etc. [12]
*Specify one more than/twice a square number, etc. [10]
*Find simple sums and products, etc. [11]
*Find sums/differences of squares, cubes, etc. [10]
*How to prove even, odd, remainder, positive, etc. [13]
Find a counterexample
*Primes, factors, inequalities [11]
*Formulae for primes, squares, cubes [11]
*Quadrilateral facts [10]
Algebraic proof
*Combining evens and odds [14]
*Multiples and remainders [15]
*Primes, factors, inequalities [11]
*Formulae for primes, squares, cubes [12]
*Quadrilateral facts [10]
*Algebraic exhaustion, factorials [8]
*Completing the square
*Misc proof
True or false?
*Primes, factors, inequalities
*Formulae for primes, squares, cubes
*Quadrilateral facts

PURE: THE BINOMIAL EXPANSION

Pascal's triangle, factorials and nCr
Pascal's triangle
*Complete the given row [5]
*Find the next two values in the given row [10]
Evaluate factorial expressions
*Evaluate n!
*Evaluate m!/n!
*Evaluate algebraic n!/(n-k)! or (n+k)!/n!
Evaluate nCr
*Evaluate small numerical nCr
*Evaluate larger numerical nCr
*Evaluate algebraic nCr [8]

Binomial expansions
$(1 \pm bx)^n$
*Expand $(1 + x)^n$
*Expand $(1 + bx)^n$
*Expand $(1 - bx)^n$
*Expand $(1 \pm bx)^n$
*Expand $(1 + x^2)^n$, etc.
$(a \pm bx)^n$
*Expand $(a + bx)^n$
*Expand $(a - bx)^n$
*Expand $(a \pm bx)^n$
$(x \pm by)^n$
*Expand $(x + y)^n$
*Expand $(x + by)^n$
*Expand $(x - by)^n$
*Expand $(x \pm by)^n$
$(ax \pm by)^n$
*Expand $(ax + by)^n$
*Expand $(ax - by)^n$
*Expand $(ax \pm by)^n$

PURE: TRIGONOMETRIC RATIOS

Non-right-angled triangles
Find a missing side or angle
*Sine Rule - find Side
*Sine Rule - find Angle
*Sine Rule - find Misc
*Cosine Rule - find Side
*Cosine Rule - find Angle
*Cosine Rule - find Misc

*Sine/Cosine Rule - find Side
*Sine/Cosine Rule - find Angle
*Sine/Cosine Rule - find Misc

Find the area

*Find area of triangle

Trigonometric graphs

Sketch a graph

*y=sinx, cosx, tanx [3]
*y=cosecx, secx, cotx [3]
*Trig graph [6]
*$y=\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ [3]

PURE: TRIG EQUATIONS

Exact values of sin/cos/tan

0 to 90°

*sin 0, 30, 45, 60, 90° [5]
*cos 0, 30, 45, 60, 90° [5]
*tan 0, 30, 45, 60, 90° [5]
*sin/cos/tan 0, 30, 45, 60, 90°

120 to 360°

*sin 120-360° [12]
*cos 120-360° [12]
*tan 120-360° [12]
*sin/cos/tan 120-360°

0 to -360°

*sin 0 to -360° [17]
*cos 0 to -360° [17]
*tan 0 to -360° [17]
*sin/cos/tan 0 to -360°

Solve trig equations: exact solutions

sin/cos/tan x = c

*Solve sin x = c
*Solve cos x = c
*Solve tan x = c
*Solve sin/cos/tan x = c

sin/cos/tan kx = c

*Solve sin kx = c
*Solve cos kx = c
*Solve tan kx = c
*Solve sin/cos/tan kx = c

PURE: DIFFERENTIATION

Differentiation basics

Definitions

*Definition to term [5]
*Term to definition [5]

Differentiation by rule

*$y = x^n$
*$y = Ax^n$
*$y = Ax^{-n}$
*$y = A/x^n$
*$y = Ax^n$, $y = Ax^{-n}$ or $y = A/x^n$
*$y = 5x^3 - 3x^2 + 6$, etc.
*$y = 5x^3 - 3/x^2$, etc.

Applying differentiation

Gradient at a point

*$y = 5x^3$, etc.
*$y = 4/x$, etc.
*$y = 5x^3$, $y = 4/x$, etc.
*$y = 5x^3 - 3x^2 + 6$, etc.
*$y = 5x^3 - 3/x^2$, etc.

Solve gradient = m

*$y = 5x^3$, etc.
*$y = 4/x$, etc.
*$y = 5x^3$, $y = 4/x$, etc.

Find maximum/minimum points

*Quadratics
*Cubics
*Quadratics/cubics

Kinematics

*Find v given $x=at^3+bt^2+ct+d$
*Find a given $x=at^3+bt^2+ct+d$
*Find a given $v=at^3+bt^2+ct+d$
*Find v or a given x or v
*Find v at t=k given $x=at^3+bt^2+ct+d$
*Find a at t=k given $x=at^3+bt^2+ct+d$
*Find a at t=k given $v=at^3+bt^2+ct+d$
*Find v or a at t=k given x or v

Chain rule: linear inner function

Polynomials

*$y = (2x + 7)^3$
*$y = 4(2x + 7)^3$
*$y = (2x + 7)^{-3}$
*$y = 4(2x + 7)^{-3}$
*$y = 1/(2x + 7)^3$
*$y = 4/(2x + 7)^3$

Exponentials

*$y = e^{3x}$, etc.
*$y = 4e^{3x}$, etc.

Trig functions

*$y = 4\sin(3x)$, etc.
*$y = 4\cos(3x)$, etc.
*$y = 4\tan(3x)$, etc.
*Misc trig function

PURE: INTEGRATION

Indefinite integration Ax^n

Given dy/dx, find y

*x^3, etc.
*Ax^3, etc.
*Ax^{-3}, etc.
*Ax^3 or Ax^{-3}, etc.
*A/x^3, etc.

*$Ax^{1/3}$, etc.
*$A\sqrt[3]{x}$, etc.
*$Ax^{-1/3}$, etc.
*$5x^3 - 3x^2 + 6$, etc.
*$5x^3 - 3/x^2$, etc.
Integrate
*x^3, etc.
*Ax^3, etc.
*Ax^{-3}, etc.
*Ax^3 or Ax^{-3}, etc.
*A/x^3, etc.
*$Ax^{1/3}$, etc.
*$A\sqrt[3]{x}$, etc.
*$Ax^{-1/3}$, etc.
*$5x^3 - 3x^2 + 6$, etc.
*$5x^3 - 3/x^2$, etc.
*$x^3(3x^2 + 4)$, etc.
*$(2x - 3)(4x + 1)$, etc.
*$(2x - x^3)(3x^2 + 6)$, etc.
*Misc product
*$(3x^3 + 4x^2)/x$, etc.

Definite integration Ax^n
Evaluate with +ve limits
*x^3, etc.
*Ax^3, etc.
*Ax^{-3}, etc.
*Ax^3 or Ax^{-3}, etc.
*A/x^3, etc.
*$Ax^{1/3}$, etc.
*$A\sqrt[3]{x}$, etc.
*$Ax^{-1/3}$, etc.
*$5x^3 - 3x^2 + 6$, etc.
*$5x^3 - 3/x^2$, etc.
*$x^3(3x^2 + 4)$, etc.
*$(2x - 3)(4x + 1)$, etc.
*$(2x - x^3)(3x^2 + 6)$, etc.
*Misc product
*$(3x^3 + 4x^2)/x$, etc.
Evaluate with ±ve limits
*x^3, etc.
*Ax^3, etc.
*Ax^{-3}, etc.
*Ax^3 or Ax^{-3}, etc.
*A/x^3, etc.
*$x^3(3x^2 + 4)$, etc.
*$(2x - 3)(4x + 1)$, etc.
*$(2x - x^3)(3x^2 + 6)$, etc.
*Misc product
*$(3x^3 + 4x^2)/x$, etc.

PURE: EXPONENTIALS and LOGARITHMS

Basics of logarithms
Rewrite logs and powers
*$3^2 = 9$, etc. to log form
*$\log_4 64 = 3$, etc. to power form
Evaluate logs
*$\log_3 81$, etc.
*$\log_3 1/81$, etc.
*$\log_3 \sqrt{3}$, etc.
*$\log_{1/4} 16$, etc.
*Misc exact logs
*$\log_n n^3$, $\log_n 1/\sqrt{n}$, etc.
*Calculator: $\log_3 4$, etc.
Log laws: numeric combine
*$\log_3 6 + \log_3 8$
*$\log_3 2 + \log_3 4 + \log_3 5$
*$\log_3 8 - \log_3 4$
*$\log_3 8 + \log_3 5 - \log_3 2$
*Misc log add/subtract
*$2\log_3 4$
*Misc basic log laws
*$3\log_3 4 + \log_3 2$
*$3\log_3 4 - \log_3 2$
*$3\log_3 4 \pm \log_3 2$
*Misc combining logs
Log laws: algebraic combine
*$2\log_{10} x + \log_{10} y - 3\log_{10} z$
*$2\log_{10} x + \log_{10} y - 3\log_{10} z + 4$
*$2\log_{10} x + \frac{1}{2}\log_{10} y - 3\log_{10} z$
*$2\log_{10} x + \frac{1}{2}\log_{10} y - 3\log_{10} z - 4$
*$2\log_n x + \log_n y - 3\log_n z$
*$2\log_n x + \log_n y - 3\log_n z + 4$
*$2\log_n x + \frac{1}{2}\log_n y - 3\log_n z$
*$2\log_n x + \frac{1}{2}\log_n y - 3\log_n z - 4$
Log laws: split
*$\log_{10} (x^2y/z^3)$
*$\log_{10} (1000x^2y/z^3)$
*$\log_{10} (x^2\sqrt{y}/z^3)$
*$\log_{10} (x^2\sqrt{y}/1000z)$
*$\log_n (x^2y/z^3)$
*$\log_n (n^3x^2y/z^3)$
*$\log_n (x^2\sqrt{y}/z^3)$
*$\log_n (x^2\sqrt{y}/n^3z)$

Equations with logarithms
Solve log equations
*$\log_3 x = 4$, etc.
*$\log_{1/3} x = 4$, etc.
*$\log_3 x = 4$, $\log_{1/3} x = 4$, etc.
*$\log_x 81 = 2$, etc.
Solve equations by taking logs

*$3^x = 7$, etc.
*$2^{3x+4} = 7$, etc.

Exponentials
Differentiate
*$y = e^{3x}$
*$y = 4e^{3x}$

PURE: FURTHER MATHS

FM Matrices
Basic operations (2x2)
*A + B
*A - B
*kA
*kA + lB
*kA - lB
*AB
*A^2
*Det(A)
*Is A singular / Det(A)=0 ?
*A^{-1}
Solve matrix and vector equations (2x2)
*Solve AX = B for matrix X
*Solve XA = B for matrix X
Basic operations (3x3)
*A + B
*A - B
*kA
*kA + lB
*kA - lB
*AB
*A^2
*Det(A)
*Is A singular / Det(A)=0 ?
*Cofactors of A
*Transpose of cofactors of A
*A^{-1}

FM Complex Numbers
Basic operations: a+bi
*Real part of a complex number
*Imaginary part of a complex number
*Add two complex numbers
*Subtract two complex numbers
*Multiply two complex numbers
*Square a complex number
*Conjugate of a complex number
*Multiply a complex number by its conjugate
*Reciprocal of a complex number
*Divide two complex numbers
*Modulus of a complex number
*Argument of a complex number
Quadratics with complex roots
*Solve $z^2 + c = 0$ (integer)
*Solve $z^2 + c = 0$ (surd)
*Solve $z^2 = a + bi$ (complex z)
*Solve $z^2 + bz + c = 0$ (conjugate pair)
*Solve $z^2 + bz + c = 0$ (non conjugate pair)

FM Vectors
Scalar (dot) product
*Scalar product of two vectors
*Are two vectors perpendicular?
*Is angle ABC a right angle?
*Magnitude of a vector
*Angle between two vectors
*Acute angle between two vectors
*Angle ABC
*Make a perpendicular vector
Vector (cross) product
*Vector product of two vectors
*Unit normal vector
Area and Volume using vectors
*Area of parallelogram given 2 sides
*Area of parallelogram given 4 points
*Area of triangle given 2 sides
*Area of triangle given 3 points
*Volume of parallelepiped given 3 edges
*Volume of tetrahedron given 3 edges
*Volume of tetrahedron given 4 points
Lines in 3D
*Vector to Cartesian equation of a line (standard)
*Cartesian to vector equation of a line (standard)
*Vector to Cartesian equation of a line (harder)
*Cartesian to vector equation of a line (harder)
*Vector equation of a line through 2 points
*Cartesian equation of a line through 2 points
*Acute angle between two lines (vector/vector)
*Acute angle between two lines (vector/Cartesian)
*Acute angle between two lines (Cartesian/Cartesian)
*Does a point lie on a line (vector)?
*Does a point lie on a line (Cartesian)?
*Do three points lie on the same line?
*Perpendicular distance from a point to a line (vector)
*Perpendicular distance from a point to a line (Cartesian)
*Two lines (vector/vector): intersect/skew/parallel?
*Two lines (vector/Cartesian): intersect/skew/parallel?
*Two lines (Cartesian/Cartesian): intersect/skew/parallel?
*Shortest distance between two lines (vector)
*Shortest distance between two lines (Cartesian)

Planes in 3D
*Vector to Cartesian equation of a plane
*Vector equation of a plane through 3 points
*Cartesian equation of a plane through 3 points
*Vector equation of a plane containing a point and a line
*Cartesian equation of a plane containing a point and a line
*Acute angle between two planes (Cartesian)
*Acute angle between a plane (Cartesian) and a line
*Perpendicular distance of a plane (Cartesian) from the origin
*Perpendicular distance of a plane (Cartesian) from a point P
*Does a point lie in a plane (Cartesian)?
*Do four points lie in the same plane?
*Vector line of intersection of two planes (Cartesian)
*Point of intersection of a line (vector) with a plane (Cartesian)
*Point of intersection of a line (Cartesian) with a plane (Cartesian)

A LEVEL STATISTICS

Data Collection
Types of data
*Qualitative and Quantitative
*Discrete and Continuous
Sampling Methods
*Census and Sample [9]
*Define sampling methods [5]
*Name sampling methods [5]
*Advantages and disadvantages [10]

Median, Quartiles from raw data
List of integers
*Median
*Lower Quartile
*Upper Quartile
*IQR
*Median/quartiles
List of decimals
*Median
*Lower Quartile
*Upper Quartile
*IQR
*Median/quartiles
Outliers
*Check one value using 1.5×IQR: integer
*Check one value using 1.5×IQR: decimal
*Find outliers using 1.5×IQR: integer list
*Find outliers using 1.5×IQR: decimal list
*Find extreme outliers using 3×IQR: integer list
*Find extreme outliers using 3×IQR: decimal list
Box Plots
*Find median
*Find quartiles
*Find IQR
*Find range
*Find misc
*Draw from median, LQ, UQ, min, max
*Draw from median, LQ, IQR, min, max
*Draw from median, LQ, UQ, min, range
*Draw from median, IQR, UQ, range, max
*Draw from misc
*Draw including an outlier
*Compare two box plots
Discrete frequency table
*Median
*Lower Quartile
*Upper Quartile
*IQR
*Median/quartiles
Grouped frequency table interpolate
*Median

*Lower Quartile
*Upper Quartile
*IQR
*Median/quartiles
Grouped frequency interpolate with rounding
*Median
*Lower Quartile
*Upper Quartile
*IQR
*Median/quartiles

Mean, Variance, SD
List of integers
*Mean
*Variance
*Standard Deviation
*Mean/var/SD
*Variance/SD
*$\sum x$
*$\sum x^2$
List of decimals
*Mean
*Variance
*Standard Deviation
*Mean/var/SD
*Variance/SD
*$\sum x$
*$\sum x^2$
Outliers
*Check one value using 2×SD: integer
*Check one value using 2×SD: decimal
*Find outliers using 2×SD: integer list
*Find outliers using 2×SD: decimal list
Discrete frequency table
*Mean
*Variance
*Standard Deviation
*Mean/var/SD
*Variance/SD
*$\sum fx$
*$\sum fx^2$
Grouped frequency table
*Mean
*Variance
*Standard Deviation
*Mean/var/SD
*Variance/SD
*$\sum fx$
*$\sum fx^2$
Grouped frequency with rounding
*Mean
*Variance
*Standard Deviation
*Mean/var/SD
*Variance/SD
*$\sum fx$
*$\sum fx^2$
Using $\sum x$, $\sum x^2$
*Find mean
*Find variance
*Find SD
*Find variance from mean, $\sum x^2$
*Find SD from mean, $\sum x^2$
*Find mean from variance, $\sum x^2$
*Find mean from SD, $\sum x^2$
*Find n from mean, variance, $\sum x^2$
*Find n from variance, $\sum x$, $\sum x^2$
*Find misc quantity
Using $\sum fx$, $\sum fx^2$
*Find mean
*Find variance
*Find SD
*Find variance from mean, $\sum fx^2$
*Find SD from mean, $\sum fx^2$
*Find mean from variance, $\sum fx^2$
*Find mean from SD, $\sum fx^2$
*Find n from mean, variance, $\sum fx^2$
*Find n from variance, $\sum fx$, $\sum fx^2$
*Find misc quantity
Adding an extra data point
*Find new mean using $\sum x$
*Find new mean using old mean
*Find value to make target mean
*Find new variance using $\sum x$, $\sum x^2$
*Find new variance using old mean, variance
*Find new SD using $\sum x$, $\sum x^2$
*Find new SD using old mean, SD
*Find misc quantity
Combining $\sum fx$, $\sum fx^2$
*Find new mean using $\sum x$
*Find new mean using old means
*Find new variance using $\sum x$, $\sum x^2$
*Find new variance using old means, variances
*Find new SD using $\sum x$, $\sum x^2$
*Find new SD using old means, SDs
*Find misc quantity
Coding: effect of transformations
*Translation and the mean
*Translation and the variance
*Translation and the SD
*Translation and misc quantity
*Scaling and the mean
*Scaling and the variance
*Scaling and the SD
*Scaling and misc quantity
*Translation & scaling and the mean
*Translation & scaling and the variance

*Translation & scaling and the SD
*Translation & scaling and misc quantity
Coding: y=(x-a)/b: x to y
*Code data values
*Find coded mean
*Find coded variance
*Find coded standard deviation
*Find misc coded quantity
Coding: y=(x-a)/b: y to x
*Uncode data values
*Find uncoded mean
*Find uncoded variance
*Find uncoded standard deviation
*Find misc uncoded quantity

Binomial probability

Rewrite using P(X = k)
*P(X < k)
*P(X > k)
*P(X = k)
*P(X </>/= k)
*P(k1 < X < k2)
*P(k1 = X < k2)
*P(k1 < X = k2)
*P(k1 = X = k2)
*P(k1 </= X </= k2)
*Misc
Find a probability from B(N, p)
*P(X = k)
*P(X < k)
*P(X = k)
*P(X > k)
*P(X = k)
*P(X </=/>/= k)
*P(m < X < n)
*P(m = X < n)
*P(m < X = n)
*P(m = X = n)
*P(m </= X </= n)
*Misc cumulative
Find a probability from N & p
*Prob of k successes
*Prob of < k successes
*Prob of = k successes
*Prob of > k successes
*Prob of = k successes
*Prob of </=/>/= k successes
*Prob of m < X < n successes
*Prob of m = X < n successes
*Prob of m < X = n successes
*Prob of m = X = n successes
*Prob of m </= X </= n successes
*Misc cumulative
Find a probability from N & q
*Prob of k successes
*Prob of < k successes
*Prob of = k successes
*Prob of > k successes
*Prob of = k successes
*Prob of </=/>/= k successes
*Prob of m < X < n successes
*Prob of m = X < n successes
*Prob of m < X = n successes
*Prob of m = X = n successes
*Prob of m </= X </= n successes
*Misc cumulative
Find a probability (N biased coins)
*Prob of k Heads
*Prob of < k Heads
*Prob of = k Heads
*Prob of > k Heads
*Prob of = k Heads
*Prob of </=/>/= k Heads
*Prob of m < X < n Heads
*Prob of m = X < n Heads
*Prob of m < X = n Heads
*Prob of m = X = n Heads
*Prob of m </= X </= n Heads
*Misc cumulative
Find a probability (N 6-sided dice)
*Prob of k successes
*Prob of < k successes
*Prob of = k successes
*Prob of > k successes
*Prob of = k successes
*Prob of </=/>/= k successes
*Prob of m < X < n successes
*Prob of m = X < n successes
*Prob of m < X = n successes
*Prob of m = X = n successes
*Prob of m </= X </= n successes
*Misc cumulative
Find a probability (N 10-sided dice)
*Prob of k successes
*Prob of < k successes
*Prob of = k successes
*Prob of > k successes
*Prob of = k successes
*Prob of </=/>/= k successes
*Prob of m < X < n successes
*Prob of m = X < n successes
*Prob of m < X = n successes
*Prob of m = X = n successes
*Prob of m </= X </= n successes
*Misc cumulative

Binomial hypothesis testing

Find the critical region

*1-tailed test; lower tail
*1-tailed test; upper tail
*2-tailed test; below sig level
*2-tailed test; closest to sig level
*Misc 1- or 2-tailed test

Find the acceptance region
*1-tailed test; lower tail
*1-tailed test; upper tail
*2-tailed test; below sig level
*2-tailed test; closest to sig level
*Misc 1- or 2-tailed test

Is value in critical region?
*1-tailed test; lower tail
*1-tailed test; upper tail
*2-tailed test; below sig level
*2-tailed test; closest to sig level
*Misc 1- or 2-tailed test

Do full hypothesis test
*1-tailed test; lower tail
*1-tailed test; upper tail
*2-tailed test; below sig level
*2-tailed test; closest to sig level
*Misc 1- or 2-tailed test

Find actual significance level
*1-tailed test; lower tail
*1-tailed test; upper tail
*2-tailed test; below sig level
*2-tailed test; closest to sig level
*Misc 1- or 2-tailed test

Normal Distribution

Basics
*Features of a Normal distribution [6]
*Recall 'famous' values ±s, ±2s, ±3s

Standard Normal (Z) Distribution N(0, 1)
*Find P(Z = k)
*Find P(Z = k)
*Find P(m = Z = n)
*Find misc P

General Normal Distribution N(μ, s^2)
*Find P(Z = k)
*Find P(Z = k)
*Find P(m = Z = n)
*Find misc P
*Wordy P(Z = k)
*Wordy P(Z = k)
*Wordy P(m = Z = n)
*Wordy P(within k of μ)
*Find misc wordy P

Inverse Standard Normal N(0, 1)
*Solve P(Z = ?) = k
*Solve P(Z = ?) = k
*Solve P(m = Z = ?) = k
*Solve P(? = Z = n) = k
*Solve P(-? = Z = ?) = k
*Solve misc P

Inverse General Normal N(μ, s^2)
*Solve P(Z = ?) = k
*Solve P(Z = ?) = k
*Solve P(m = Z = ?) = k
*Solve P(? = Z = n) = k
*Solve misc P
*Wordy solve P(Z = ?) = k
*Wordy solve P(Z = ?) = k
*Wordy solve P(m = Z = ?) = k
*Wordy solve P(? = Z = n) = k
*Solve misc wordy P

Find values of μ, s
*Find μ given P(X = k)
*Find μ given P(X = k)
*Find s given P(X = k)
*Find s given P(X = k)
*Find misc μ, s
*Find μ & s given P(X = m), P(X = n)
*Find μ & s given P(X = m), P(X = n)
*Find misc μ & s
*Wordy find μ given P(X = k)
*Wordy find μ given P(X = k)
*Wordy find s given P(X = k)
*Wordy find s given P(X = k)
*Find misc wordy μ, s
*Wordy find μ & s given P(X = m), P(X = n)
*Wordy find μ & s given P(X = m), P(X = n)
*Find misc wordy μ & s

Normal hypothesis tests of a population mean

Find the critical region
*1-tailed test; lower tail
*1-tailed test; upper tail
*2-tailed test
*Misc 1- or 2-tailed test

Find the acceptance region
*1-tailed test; lower tail
*1-tailed test; upper tail
*2-tailed test
*Misc 1- or 2-tailed test

Is value in critical region?
*1-tailed test; lower tail
*1-tailed test; upper tail
*2-tailed test
*Misc 1- or 2-tailed test

Do full hypothesis test
*1-tailed test; lower tail
*1-tailed test; upper tail
*2-tailed test
*Misc 1- or 2-tailed test

Normal approximation to Binomial

Basics

*Is the Normal approximation valid?
*Find mean and variance from B(n, p)
Continuity corrections
*Binomial P(X = b) to Normal
*Binomial P(X = b) to Normal
*Binomial P(X < b) to Normal
*Binomial P(X = b) to Normal
*Binomial P(X > b) to Normal
*Misc Binomial to Normal (1)
*Binomial P(a = X = b) to Normal
*Binomial P(a = X < b) to Normal
*Binomial P(a < X = b) to Normal
*Binomial P(a < X < b) to Normal
*Misc Binomial to Normal (2)
Find probabilities
*Find Binomial P(X = b) using Normal
*Find Binomial P(X = b) using Normal
*Find Binomial P(X < b) using Normal
*Find Binomial P(X = b) using Normal
*Find Binomial P(X > b) using Normal
*Find misc Binomial using Normal (1)
*Find Binomial P(a = X = b) using Normal
*Find Binomial P(a = X < b) using Normal
*Find Binomial P(a < X = b) using Normal
*Find Binomial P(a < X < b) using Normal
*Find misc Binomial using Normal (2)

Correlation

Definitions
*Correlation terms [8]
*Regression terms [8]
Scatter diagrams & PMCC
*Describe type of correlation
*Estimate the PMCC
*Calculate PMCC from table
Hypothesis testing with PMCC
*1-tailed +ve: find critical region
*1-tailed -ve: find critical region
*2-tailed: find critical region
*Misc find critical region
*1-tailed +ve: is value in critical region?
*1-tailed -ve: is value in critical region?
*2-tailed: is value in critical region?
*Misc is value in critical region?
*1-tailed +ve: hypothesis test
*1-tailed -ve: hypothesis test
*2-tailed: hypothesis test
*Misc hypothesis test
Coding exponential models
*$y = a x^n$ from linear
*$y = k b^x$ from linear
*$y = a x^n$ or $y = k b^x$ from linear
*$y = a x^n$ from gradient & y-int
*$y = k b^x$ from gradient & y-int
*$y = a x^n$ or $y = k b^x$ from gradient & y-int
*$y = a x^n$ to linear
*$y = k b^x$ to linear
*$y = a x^n$ or $y = k b^x$ to linear

Probability: Venn diagrams

Interpret
*2-Venn: find P(A and not B), etc. [10]
*3-Euler: find P(A and not B), etc. [24]
*3-Venn: find P(A and not B), etc. [24]
*2-Venn: find P(A ∩ B'), etc. [10]
*3-Euler: find P(A ∩ B'), etc. [24]
*3-Venn: find P(A ∩ B'), etc. [24]
Fill in frequencies
*2-Venn, eg n(A or B)
*3-Euler, eg n(A or B)
*3-Venn, eg n(A or B)
*Misc Venn, eg n(A or B)
*2-Venn, eg n(A U B)
*3-Euler, eg n(A U B)
*3-Venn, eg n(A U B)
*Misc Venn, eg n(A U B)
*2-Venn wordy
*3-Euler wordy
*3-Venn wordy
*Misc Venn wordy
Fill in probabilities
*2-Venn, eg p(A or B)
*3-Euler, eg p(A or B)
*3-Venn, eg p(A or B)
*Misc Venn, eg p(A or B)
*2-Venn, eg p(A U B)
*3-Euler, eg p(A U B)
*3-Venn, eg p(A U B)
*Misc Venn, eg p(A U B)
Independence: all values given
*2-Venn: are A and B independent?
*2-Venn: are A' and B independent?
*2-Venn: are A and B' independent?
*2-Venn: are A' and B' independent?
*2-Venn: are A/A' and B/B' independent?
*3-Venn: are A and B independent?
*3-Venn: are A' and B independent?
*3-Venn: are A and B' independent?
*3-Venn: are A' and B' independent?
*3-Venn: are A/A' and B/B' independent?
Independence: one blank value
*2-Venn: are A and B independent?
*2-Venn: are A' and B independent?
*2-Venn: are A and B' independent?
*2-Venn: are A' and B' independent?
*2-Venn: are A/A' and B/B' independent?
*3-Venn: are A and B independent?
*3-Venn: are A' and B independent?

*3-Venn: are A and B' independent?
*3-Venn: are A' and B' independent?
*3-Venn: are A/A' and B/B' independent?
Find x if independent
*2-Venn: one solution 1
*2-Venn: one solution 2
*2-Venn: misc one solution
*2-Venn: two solutions 1
*2-Venn: two solutions 2
*2-Venn: misc two solutions
*2-Venn: misc 1/2 solutions
Find x if mutually exclusive
*2-Venn: one solution 1
*2-Venn: one solution 2
*2-Venn: misc one solution
Conditional probability
*2-Venn: find P(A | A U B), etc. [16]
*3-Euler: find P(A | A U B), etc. [32]
*3-Venn: find P(A | A U B), etc. [32]

Probability: two-way tables

Interpret
*2-way table: find P(A and not B), etc. [10]
*2-way table: find P(A ∩ B'), etc. [10]
Fill in frequencies
*2-way table, eg n(A or B)
*2-way table, eg n(A U B)
*2-way table wordy
Fill in probabilities
*2-way table, eg p(A or B)
*2-way table, eg p(A U B)
Independence: all values given
*2-way table: are A and B independent?
*2-way table: are A' and B independent?
*2-way table: are A and B' independent?
*2-way table: are A' and B' independent?
*2-way table: are A/A' and B/B' independent?
Independence: one blank value
*2-way table: are A and B independent?
*2-way table: are A' and B independent?
*2-way table: are A and B' independent?
*2-way table: are A' and B' independent?
*2-way table: are A/A' and B/B' independent?
Find x if independent
*2-way table: one solution 1
*2-way table: one solution 2
*2-way table: misc one solution
*2-way table: two solutions 1
*2-way table: two solutions 2
*2-way table: misc two solutions
*2-way table: misc 1/2 solutions
Find x if mutually exclusive
*2-way table: one solution 1
*2-way table: one solution 2
*2-way table: misc one solution
Conditional probability
*2-way table: find P(A | A U B), etc. [16]

NETWORKS/DECISION

Networks

Minimum Spanning Tree
*Prim
*Kruskal
*Matrix Prim
Shortest Route
*Dijkstra
Route Inspection
*Best pairing of 4 odd nodes
*Route Inspection - 4 odd nodes
Travelling Salesperson
*Nearest neighbour (single)
*Lower bound (single)
*Nearest neighbour (all nodes)
*Lower bound (all nodes)

Algorithms

Packing
*First fit
*First fit decreasing
*Full bin
Sorting
*Bubble sort
*Shuttle sort
Order of Algorithms
*Find order from efficiency
*Calculate using order

Linear Programming

LP Graphical Solution
*Terminology
*Definitions
*Solving from graph
*Solving from LP formulation
LP Simplex Method
*2-variable simplex
*3-variable simplex

Circle Equations

MATHSprint

Answer on file paper

1: Work out the following:

a) The equation of a circle centre (−5, 2) and radius 12.

b) The equation of a circle centre (−3, −6) and radius 10.

2: Work out the following:

a) The equation of a circle centre (−1, 3) passing through the point (3, 9).

b) The equation of a circle centre (−4, 0) passing through the point (−3, 5).

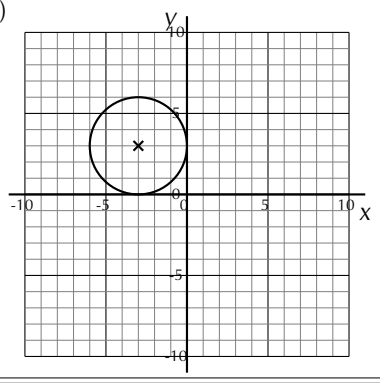
3: Work out the following:

a) The equation of a circle with diameter from (−5, 0) to (1, −10).

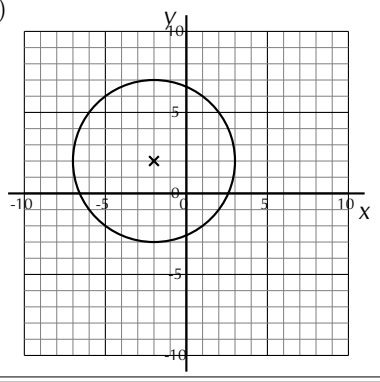
b) The equation of a circle with diameter from (−7, 2) to (1, 6).

4: Give an equation for each of the following in the form $(x - a)^2 + (y - b)^2 = r^2$

a)

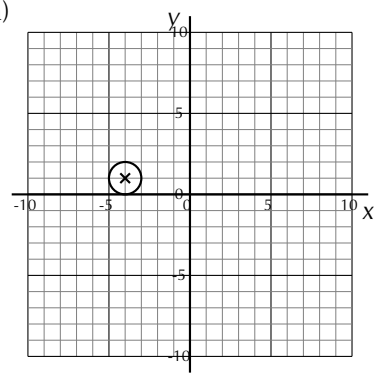


b)

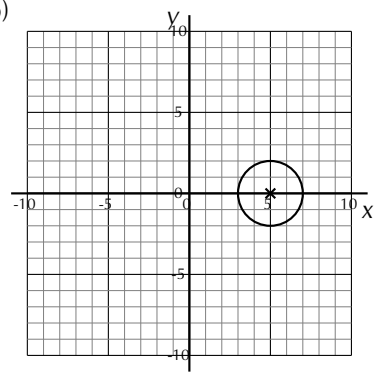


5: Give an equation for each of the following in the form $x^2 + y^2 + fx + gy + c = 0$

a)



b)



6: Find the co-ordinates of the points where the following intersect:

a) The circle $(x + 4)^2 + (y + 3)^2 = 13$ and the x-axis.

b) The circle $(x + 1)^2 + (y - 1)^2 = 10$ and the y-axis.

7: How many times do the following intersect?

a) The circle $(x - 3)^2 + (y - 1)^2 = 5$ and the line $x + 2y - 7 = 0$.

b) The circle $x^2 + (y - 4)^2 = 20$ and the line $x - 2y - 2 = 0$.

8: Find the coordinates of the point(s) of intersection of the following:

a) $y = -2x + 6$
$(x + 3)^2 + (y - 2)^2 = 20$

b) $y = x + 3$
$(x + 2)^2 + (y + 1)^2 = 20$

9: Find the (range of) values of k in each case:

a) If $x - 2y + k = 0$ does not intersect $(x - 1)^2 + y^2 = 5$.

b) If $x + y - 3 = 0$ meets $(x - k)^2 + (y - 3)^2 = 8$ at exactly one point.

Answers: Circle Equations

MATHSprint

1: a) $(x + 5)^2 + (y - 2)^2 = 144$

b) $(x + 3)^2 + (y + 6)^2 = 100$

2: a) $r^2 = (3 - (-1))^2 + (9 - 3)^2$ so
$(x + 1)^2 + (y - 3)^2 = 52$

b) $r^2 = ((-3) - (-4))^2 + (5 - 0)^2$ so
$(x + 4)^2 + y^2 = 26$

3: a) Centre $= (\frac{1 + (-5)}{2}, \frac{-10 + 0}{2}) = (-2, -5)$
$r^2 = (\frac{1 - (-5)}{2})^2 + (\frac{-10 - 0}{2})^2 = 3^2 + (-5)^2$ so
$(x + 2)^2 + (y + 5)^2 = 34$

b) Centre $= (\frac{1 + (-7)}{2}, \frac{6 + 2}{2}) = (-3, 4)$
$r^2 = (\frac{1 - (-7)}{2})^2 + (\frac{6 - 2}{2})^2 = 4^2 + 2^2$ so
$(x + 3)^2 + (y - 4)^2 = 20$

4: a) $(x + 3)^2 + (y - 3)^2 = 9$

b) $(x + 2)^2 + (y - 2)^2 = 25$

5: a) $(x + 4)^2 + (y - 1)^2 = 1$
$x^2 + y^2 + 8x - 2y + 16 = 0$

b) $(x - 5)^2 + y^2 = 4$
$x^2 + y^2 - 10x + 21 = 0$

6: a) $(x + 4)^2 + 9 = 13$
Intersections at $(-6, 0)$ and $(-2, 0)$.

b) $1 + (y - 1)^2 = 10$
Intersections at $(0, -2)$ and $(0, 4)$.

7: a) $x = -2y + 7$
$(-2y + 4)^2 + (y - 1)^2 = 5$
$5y^2 - 18y + 12 = 0$
Discriminant is
$(-18)^2 - 4 \times 5 \times 12 = 84$
2 intersections

b) $x = 2y + 2$
$(2y + 2)^2 + (y - 4)^2 = 20$
$5y^2 = 0$
Discriminant is
$0^2 - 4 \times 5 \times 0 = 0$
1 intersection

8: a) $y = -2x + 6$
$(x + 3)^2 + (-2x + 4)^2 = 20$
$5x^2 - 10x + 5 = 0$
$x^2 - 2x + 1 = 0$
$(x - 1)^2 = 0$
$(1, 4)$.

b) $y = x + 3$
$(x + 2)^2 + (x + 4)^2 = 20$
$2x^2 + 12x = 0$
$x^2 + 6x = 0$
$x(x + 6) = 0$
$(-6, -3)$ and $(0, 3)$.

9: a) $x = 2y - k$
$(2y - k - 1)^2 + (y)^2 + \ = 5$
$5y^2 + (-4k - 4)y + (k^2 + 2k - 4) = 0$
Discriminant is:
$(-4k - 4)^2 - 4 \times 5 \times (k^2 + 2k - 4)$
$= -4k^2 - 8k + 96 < 0$
$k^2 + 2k - 24 > 0$
$(k + 6)(k - 4) > 0$
$k < -6$ or $k > 4$.

b) $y = -x + 3$
$(x - k)^2 + (-x)^2 = 8$
$2x^2 + (0 - 2k)x + k^2 - 8 = 0$
Discriminant is:
$(0 - 2k)^2 - 4 \times 2 \times (k^2 - 8)$
$= -4k^2 + 64 = 0$
$k^2 - 16 = 0$
$(k + 4)(k - 4) = 0$
$k = -4$ or $k = 4$.

Exact Trig Values

MATHSprint

Answer on file paper

1: Give the following as exact values:

a) $\tan 90°$ b) $\cos 45°$ c) $\sin 45°$ d) $\tan 0°$

e) $\cos 0°$ f) $\sin 30°$ g) $\sin 60°$ h) $\cos 90°$

2: Give the following as exact values:

a) $\sin 120°$ b) $\tan 180°$ c) $\cos 135°$ d) $\tan 135°$

e) $\sin 225°$ f) $\cos 360°$ g) $\cos 330°$ h) $\sin 150°$

3: Give the following as exact values:

a) $\cos(-150°)$ b) $\tan(-60°)$ c) $\sin(-225°)$ d) $\sin(-270°)$

e) $\cos(-360°)$ f) $\tan(-30°)$ g) $\cos(-240°)$ h) $\tan(-210°)$

4: Solve the following equations for θ in the interval $0 \leq \theta < 360°$:

a) $\tan\theta = -\sqrt{3}$ b) $\cos\theta = \frac{1}{2}$ c) $\sin\theta = \frac{1}{\sqrt{2}}$ d) $\tan\theta = -\frac{1}{\sqrt{3}}$

5: Solve the following equations for θ in the interval $0 \leq \theta < 360°$:

a) $\tan 2\theta = 1$ b) $\sin 3\theta = 1$ c) $\cos 4\theta = 0$ d) $\sin 5\theta = \frac{\sqrt{3}}{2}$

Answers: Exact Trig Values

MATHSprint

1: a) $\tan 90° = \infty$ b) $\cos 45° = \frac{1}{\sqrt{2}}$ c) $\sin 45° = \frac{1}{\sqrt{2}}$ d) $\tan 0° = 0$

e) $\cos 0° = 1$ f) $\sin 30° = \frac{1}{2}$ g) $\sin 60° = \frac{\sqrt{3}}{2}$ h) $\cos 90° = 0$

2: a) $\sin 120° = \frac{\sqrt{3}}{2}$ b) $\tan 180° = 0$ c) $\cos 135° = \frac{-1}{\sqrt{2}}$ d) $\tan 135° = -1$

e) $\sin 225° = \frac{-1}{\sqrt{2}}$ f) $\cos 360° = 1$ g) $\cos 330° = \frac{\sqrt{3}}{2}$ h) $\sin 150° = \frac{1}{2}$

3: a) $\cos(-150°) = \frac{-\sqrt{3}}{2}$ b) $\tan(-60°) = -\sqrt{3}$ c) $\sin(-225°) = \frac{1}{\sqrt{2}}$ d) $\sin(-270°) = 1$

e) $\cos(-360°) = 1$ f) $\tan(-30°) = \frac{-1}{\sqrt{3}}$ g) $\cos(-240°) = \frac{-1}{2}$ h) $\tan(-210°) = \frac{-1}{\sqrt{3}}$

4: a) $\tan\theta = -\sqrt{3}$
$\tan^{-1}(-\sqrt{3}) = -60°$
$\theta = 180 - 60 = \mathbf{120°}$
$\theta = 360 - 60 = \mathbf{300°}$

b) $\cos\theta = \frac{1}{2}$
$\cos^{-1}(\frac{1}{2}) = 60°$
$\theta = \mathbf{60°}$
$\theta = 360 - 60 = \mathbf{300°}$

c) $\sin\theta = \frac{1}{\sqrt{2}}$
$\sin^{-1}(\frac{1}{\sqrt{2}}) = 45°$
$\theta = \mathbf{45°}$
$\theta = 180 - 45 = \mathbf{135°}$

d) $\tan\theta = -\frac{1}{\sqrt{3}}$
$\tan^{-1}(-\frac{1}{\sqrt{3}}) = -30°$
$\theta = 180 - 30 = \mathbf{150°}$
$\theta = 360 - 30 = \mathbf{330°}$

5: a) $\tan 2\theta = 1,\ 0 \le \theta < 360°$
Let $X = 2\theta,\ 0 \le X < 720°$
$\tan^{-1}(1) = 45°$
$X = (45+360n)°$, $X = (225+360n)°$
$\theta = (22.5+180n)°$, $\theta = (112.5+180n)°$
$\theta = \mathbf{22.5°}$, $\mathbf{202.5°}$
$\theta = \mathbf{112.5°}$, $\mathbf{292.5°}$

b) $\sin 3\theta = 1,\ 0 \le \theta < 360°$
Let $X = 3\theta,\ 0 \le X < 1080°$
$\sin^{-1}(1) = 90°$
$X = (90+360n)°$
$\theta = (30+120n)°$
$\theta = \mathbf{30°}$, $\mathbf{150°}$, $\mathbf{270°}$

c) $\cos 4\theta = 0,\ 0 \le \theta < 360°$
Let $X = 4\theta,\ 0 \le X < 1440°$
$\cos^{-1}(0) = 90°$
$X = (90+360n)°$, $X = (270+360n)°$
$\theta = (22.5+90n)°$, $\theta = (67.5+90n)°$
$\theta = \mathbf{22.5°}$, $\mathbf{112.5°}$, $\mathbf{202.5°}$, $\mathbf{292.5°}$
$\theta = \mathbf{67.5°}$, $\mathbf{157.5°}$, $\mathbf{247.5°}$, $\mathbf{337.5°}$

d) $\sin 5\theta = \frac{\sqrt{3}}{2},\ 0 \le \theta < 360°$
Let $X = 5\theta,\ 0 \le X < 1800°$
$\sin^{-1}(\frac{\sqrt{3}}{2}) = 60°$
$X = (60+360n)°$, $X = (120 + 360n)°$
$\theta = (12+72n)°$, $\theta = (24+72n)°$
$\theta = \mathbf{12°}$, $\mathbf{84°}$, $\mathbf{156°}$, $\mathbf{228°}$, $\mathbf{300°}$
$\theta = \mathbf{24°}$, $\mathbf{96°}$, $\mathbf{168°}$, $\mathbf{240°}$, $\mathbf{312°}$

Integration

MATHSprint

Please answer in your books

1: Find an expression for y when $\frac{dy}{dx}$ is the following:

a) x^{-8} b) $6x^7$

2: Find an expression for y when $\frac{dy}{dx}$ is the following:

a) $5\sqrt[3]{x}$ b) $9x^2\sqrt{x}$

3: Find the following integrals:

a) $\int 3x^6 - 7x^4 \, dx$ b) $\int -6x^9 + 7x^5 - x^3 + 10x^2 \, dx$

4: Find the following integrals:

a) $\int x^4(x^3 + 5x) \, dx$ b) $\int x(-5x^5 + 2x^2) \, dx$

5: Find the following integrals:

a) $\int \frac{3x^7 + 4x^6}{x^4} \, dx$ b) $\int \frac{-3x^8 - 4x^5}{x^3} \, dx$

6: Evaluate the following definite integrals:

a) $\int_{1/4}^{1/2} 40x^{-5} \, dx$ b) $\int_{1}^{3} -12x^3 \, dx$

7: Evaluate the following definite integrals:

a) $\int_{1}^{625} -45\sqrt[4]{x} \, dx$ b) $\int_{64}^{125} 40(\sqrt[3]{x})^2 \, dx$

8: Evaluate the following definite integrals:

a) $\int_{0}^{5} -24x^5 + 10x^4 - 10x + 4 \, dx$ b) $\int_{1}^{5} 6x^5 - 8x^3 - 24x^2 + 10 \, dx$

9: Evaluate the following definite integrals:

a) $\int_{4}^{5} (3x - 1)(5x + 3) \, dx$ b) $\int_{1}^{2} (5x + 1)(3x - 1) \, dx$

10: Evaluate the following definite integrals:

a) $\int_{2}^{5} \frac{-5x^5 - 8x^4}{x} \, dx$ b) $\int_{3}^{5} \frac{15x^5 + 6x^3}{x} \, dx$

Answers: Integration

MATHSprint

1: a) $y = \int x^{-8}\,dx = -\frac{1}{7}x^{-7} + c$ b) $y = \int 6x^{7}\,dx = \frac{3}{4}x^{8} + c$

2: a) $y = \int 5x^{1/3}\,dx = \frac{15}{4}x^{4/3} + c$ b) $y = \int 9x^{5/2}\,dx = \frac{18}{7}x^{7/2} + c$

3: a) $\int 3x^{6} - 7x^{4}\,dx = \frac{3}{7}x^{7} - \frac{7}{5}x^{5} + c$

b) $\int -6x^{9} + 7x^{5} - x^{3} + 10x^{2}\,dx = -\frac{3}{5}x^{10} + \frac{7}{6}x^{6} - \frac{1}{4}x^{4} + \frac{10}{3}x^{3} + c$

4: a) $\int x^{7} + 5x^{5}\,dx = \frac{1}{8}x^{8} + \frac{5}{6}x^{6} + c$ b) $\int -5x^{6} + 2x^{3}\,dx = -\frac{5}{7}x^{7} + \frac{1}{2}x^{4} + c$

5: a) $\int 3x^{3} + 4x^{2}\,dx = \frac{3}{4}x^{4} + \frac{4}{3}x^{3} + c$ b) $\int -3x^{5} - 4x^{2}\,dx = -\frac{1}{2}x^{6} - \frac{4}{3}x^{3} + c$

6: a) $\int_{1/4}^{1/2} 40x^{-5}\,dx = \left[-10x^{-4} \right]_{1/4}^{1/2}$
$= (-160) - (-2560) = 2400$

b) $\int_{1}^{3} -12x^{3}\,dx = \left[-3x^{4} \right]_{1}^{3}$
$= (-243) - (-3) = -240$

7: a) $\int_{1}^{625} -45x^{1/4}\,dx = \left[-36x^{5/4} \right]_{1}^{625}$
$= (-112500) - (-36) = -112464$

b) $\int_{64}^{125} 40x^{2/3}\,dx = \left[24x^{5/3} \right]_{64}^{125}$
$= (75000) - (24576) = 50424$

8: a) $\int_{0}^{5} -24x^{5} + 10x^{4} - 10x + 4\,dx = \left[-4x^{6} + 2x^{5} - 5x^{2} + 4x \right]_{0}^{5}$
$= (-56355) - (0) = -56355$

b) $\int_{1}^{5} 6x^{5} - 8x^{3} - 24x^{2} + 10\,dx = \left[x^{6} - 2x^{4} - 8x^{3} + 10x \right]_{1}^{5}$
$= (13425) - (1) = 13424$

9: a) $\int_{4}^{5} 15x^{2} + 4x - 3\,dx = \left[5x^{3} + 2x^{2} - 3x \right]_{4}^{5}$
$= (660) - (340) = 320$

b) $\int_{1}^{2} 15x^{2} - 2x - 1\,dx = \left[5x^{3} - x^{2} - x \right]_{1}^{2}$
$= (34) - (3) = 31$

10: a) $\int_{2}^{5} -5x^{4} - 8x^{3}\,dx = \left[-x^{5} - 2x^{4} \right]_{2}^{5}$
$= (-4375) - (-64) = -4311$

b) $\int_{3}^{5} 15x^{4} + 6x^{2}\,dx = \left[3x^{5} + 2x^{3} \right]_{3}^{5}$
$= (9625) - (783) = 8842$

Name: **Class/Set:**

Logarithms

MATHSprint

1: Rewrite each statement as a logarithm:

a) $5^3 = 125$

_____

b) $4^{-4} = \frac{1}{256}$

_____

2: Rewrite each statement using a power:

a) $\log_8 \frac{1}{64} = -2$

_____

b) $\log_7 117649 = 6$

_____

3: Without using a calculator, find the value of:

a) $\log_9 9\sqrt{9}$ = _____

b) $\log_{1/7} 7$ = _____

c) $\log_9 81$ = _____

d) $\log_{12} \frac{1}{12}$ = _____

4: Without using a calculator, find the value of:

a) $\log_a \frac{1}{a^2}$ = _____

b) $\log_{1/a} \sqrt{a}$ = _____

c) $\log_{1/a} a^9$ = _____

d) $\log_a a^5$ = _____

5: Use a calculator to find the value of the following correct to 4 d.p.:

a) $\log_{11} 884$ = _____

b) $\log_{12} 1142552$ = _____

c) $\log_2 10$ = _____

d) $\log_9 66$ = _____

6: Write as a single logarithm:

a) $3\log_3 2 + \log_3 3$ = _____

b) $2\log_6 4 - \log_6 2$ = _____

c) $\log_{10} 5 + \log_{10} 6$ = _____

d) $\log_5 21 + \log_5 4 - \log_7 7$ = _____

7: Write as a single logarithm:

a) $\log_{10}x - 4\log_{10}y - 5\log_{10}z + 2$

_____

b) $-3\log_{10}x + 5\log_{10}y - 2\log_{10}z - 4$

_____

8: Write as a single logarithm:

a) $-\frac{1}{2}\log_a x - \frac{1}{3}\log_a y + 2\log_a z - 3$

_____

b) $\log_a x + \frac{1}{3}\log_a y - \frac{1}{2}\log_a z + 3$

_____

9: Write in terms of $\log_{10} x$, $\log_{10} y$ and $\log_{10} z$:

a) $\log_{10}(\sqrt{10}\sqrt[3]{x}yz^2)$

b) $\log_{10}\left(\frac{yz^2}{\sqrt{10}x^3}\right)$

10: If $p = \log_a x$, $q = \log_a y$ and $r = \log_a z$, write the following in terms of p, q and r:

a) $\log_a\left(\frac{a^2}{x^3y^5z}\right)$

b) $\log_a\left(\frac{x^3z^5}{ay^4}\right)$

11: Solve the following equations:

a) $\log_{1/3} x = -3$

b) $\log_5 x = -2$

12: Solve the following equations:

a) $\log_x 64 = 2$

b) $\log_x 36 = 2$

13: Solve the following equations correct to 3 decimal places:

a) $11^x = 96$

b) $7^x = 45$

14: Solve the following equations correct to 3 decimal places:

a) $12^{6x-4} = 28$

b) $4^{2x-2} = 40$

Answers: Logarithms

MATHSprint

1: a) $\log_5 125 = 3$ b) $\log_4 \frac{1}{256} = -4$

2: a) $8^{-2} = \frac{1}{64}$ b) $7^6 = 117649$

3: a) $\log_9 9\sqrt{9} = \frac{3}{2}$ b) $\log_{1/7} 7 = -1$ c) $\log_9 81 = 2$ d) $\log_{12} \frac{1}{12} = -1$

4: a) $\log_a \frac{1}{a^2} = -2$ b) $\log_{1/a} \sqrt{a} = -\frac{1}{2}$ c) $\log_{1/a} a^9 = -9$ d) $\log_a a^5 = 5$

5: a) $\log_{11} 884 = 2.8293$ b) $\log_{12} 1142552 = 5.6134$

c) $\log_2 10 = 3.3219$ d) $\log_9 66 = 1.9068$

6: a) $3\log_3 2 + \log_3 3 = \log_3 24$ b) $2\log_6 4 - \log_6 2 = \log_6 8$

c) $\log_{10} 5 + \log_{10} 6 = \log_{10} 30$ d) $\log_5 21 + \log_5 4 - \log_7 7 = \log_5 12$

7: a) $\log_{10}x - 4\log_{10}y - 5\log_{10}z + 2 = \log_{10}\left(\frac{100x}{y^4z^5}\right)$

b) $-3\log_{10}x + 5\log_{10}y - 2\log_{10}z - 4 = \log_{10}\left(\frac{y^5}{10000x^3z^2}\right)$

8: a) $-\frac{1}{2}\log_a x - \frac{1}{3}\log_a y + 2\log_a z - 3 = \log_a\left(\frac{z^2}{a^3\sqrt{x}\sqrt[3]{y}}\right)$ b) $\log_a x + \frac{1}{3}\log_a y - \frac{1}{2}\log_a z + 3 = \log_a\left(\frac{a^3x\sqrt[3]{y}}{\sqrt{z}}\right)$

9: a) $\log_{10}(\sqrt{10}\sqrt[3]{xyz^2}) = \frac{1}{3}\log_{10}x + \log_{10}y + 2\log_{10}z + \frac{1}{2}$

b) $\log_{10}\left(\frac{yz^2}{\sqrt{10x^3}}\right) = -3\log_{10}x + \log_{10}y + 2\log_{10}z - \frac{1}{2}$

10: a) $\log_a\left(\frac{a^2}{x^3y^5z}\right) = -3p - 5q - r + 2$ b) $\log_a\left(\frac{x^3z^5}{ay^4}\right) = 3p - 4q + 5r - 1$

11: a) $\log_{1/3} x = -3$
$x = (\frac{1}{3})^{-3} = 27$

b) $\log_5 x = -2$
$x = 5^{-2} = \frac{1}{25}$

12: a) $\log_x 64 = 2$
$x^2 = 64$ so $x = 8$

b) $\log_x 36 = 2$
$x^2 = 36$ so $x = 6$

13: a) $\log(11^x) = x\log 11 = \log 96$
so $x = \frac{\log 96}{\log 11} = 1.903$

b) $\log(7^x) = x\log 7 = \log 45$
so $x = \frac{\log 45}{\log 7} = 1.956$

14: a) $\log(12^{6x-4}) = (6x-4)\log 12 = \log 28$
so $x = \frac{1}{6}(\frac{\log 28}{\log 12} + 4) = 0.890$

b) $\log(4^{2x-2}) = (2x-2)\log 4 = \log 40$
so $x = \frac{1}{2}(\frac{\log 40}{\log 4} + 2) = 2.330$

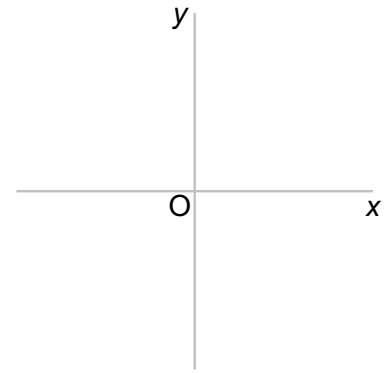
Name: **Class/Set:**

Number of solutions by sketching

MATHSprint

1: Sketch the following pairs of graphs on the same axes. Use your sketch to state the number of points of intersection of the two graphs:

a) $y = -x^3$, $y = -3x$.



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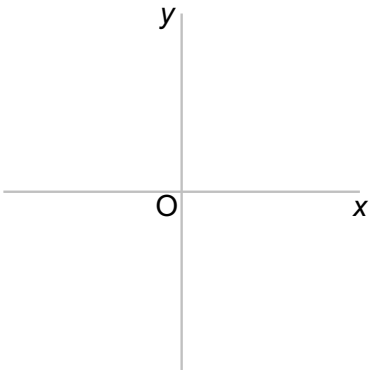
b) $y = -\frac{1}{x^2}$, $y = 2x$.



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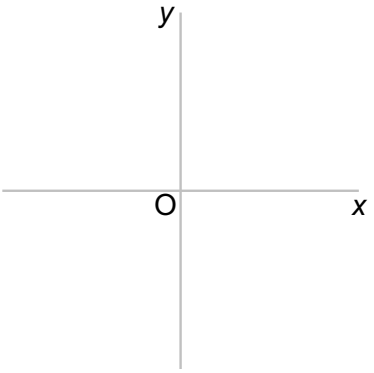
c) $y = \frac{1}{x}$, $y = -x$.



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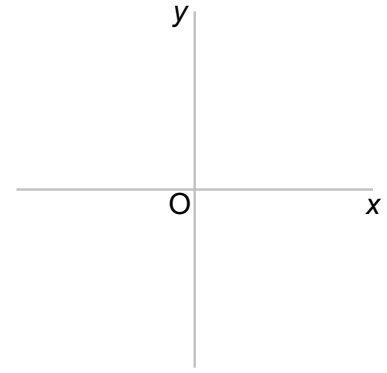
d) $y = x^2$, $y = 2$.



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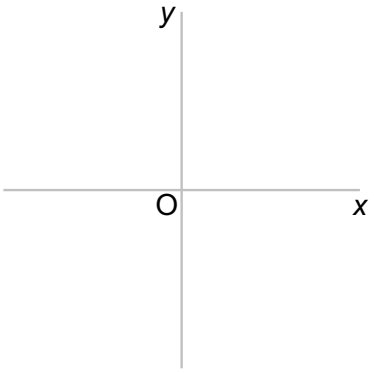
e) $y = \frac{1}{x^2}$, $y = 2x$.



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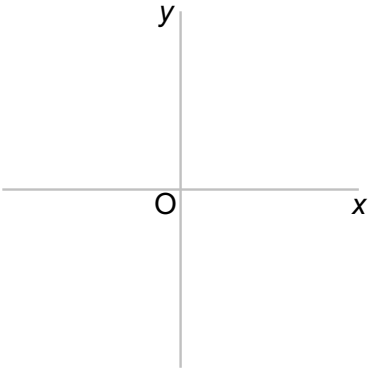
f) $y = -x(x - 1)(x + 1)$, $y = -3x$.



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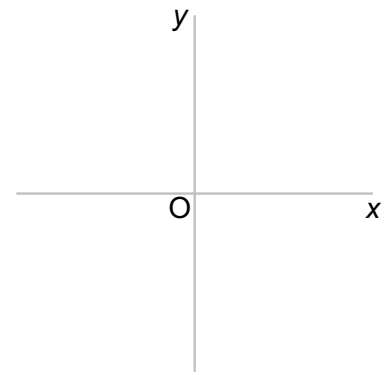
2: Sketch the following pairs of graphs on the same axes. Use your sketch to state the number of points of intersection of the two graphs:

a) $y = -\frac{1}{x^2}$, $y = -x(x+1)^2$.



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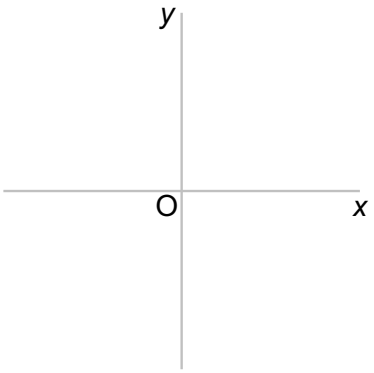
b) $y = x^3$, $y = x(x+1)$.



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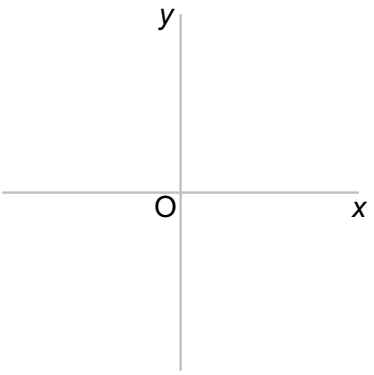
c) $y = \frac{1}{x}$, $y = -x(x+1)$.



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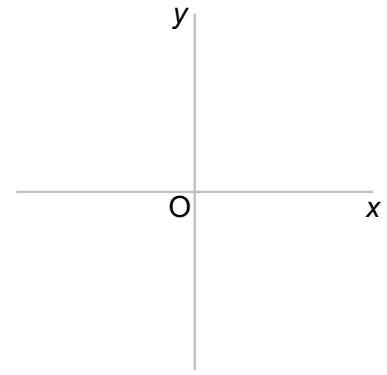
d) $y = \frac{1}{x^2}$, $y = -x^2$.



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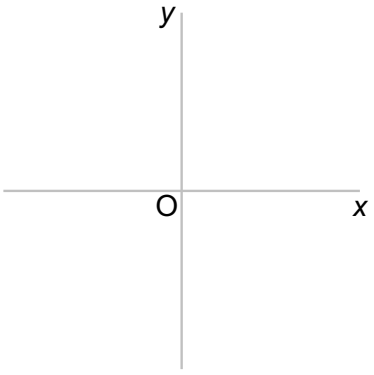
e) $y = -\frac{1}{x}$, $y = -x(x-1)(x+1)$.



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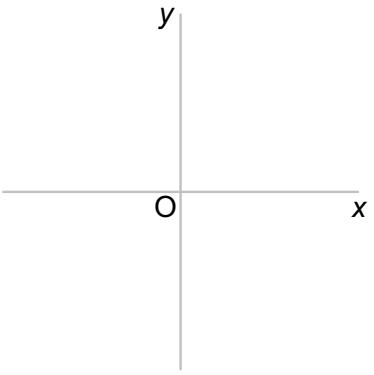
f) $y = \frac{1}{x^2}$, $y = -x(x-1)^2$.



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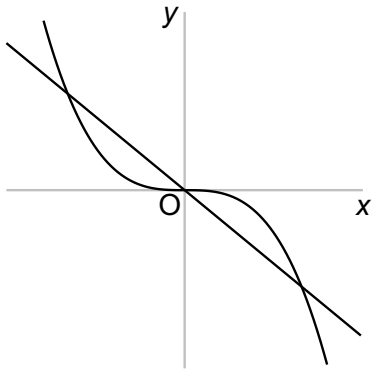
Answers: Number of solutions by sketching

MATHSprint

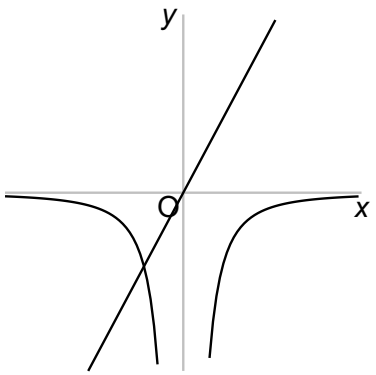
1: a) $y = -x^3$, $y = -3x$.

Number of intersections = 3.



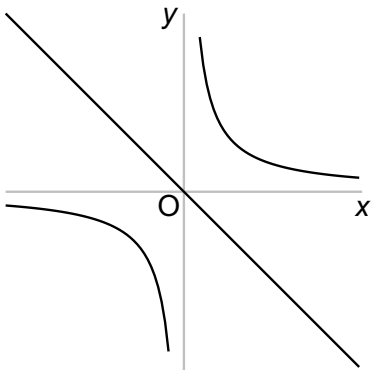
b) $y = -\frac{1}{x^2}$, $y = 2x$.

Number of intersections = 1.



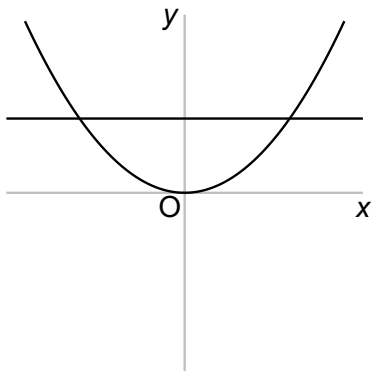
c) $y = \frac{1}{x}$, $y = -x$.

Number of intersections = 0.



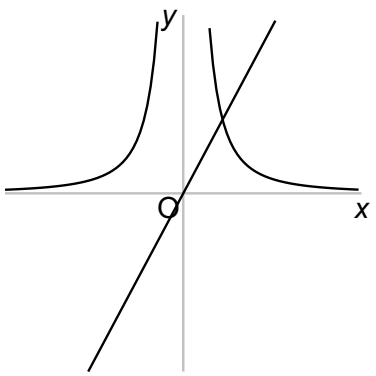
d) $y = x^2$, $y = 2$.

Number of intersections = 2.



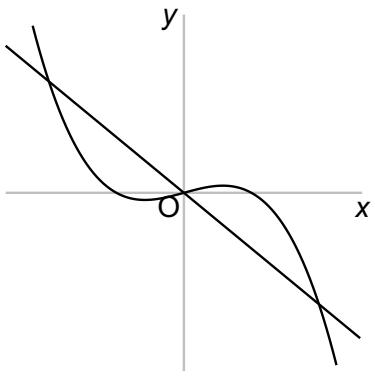
e) $y = \frac{1}{x^2}$, $y = 2x$.

Number of intersections = 1.



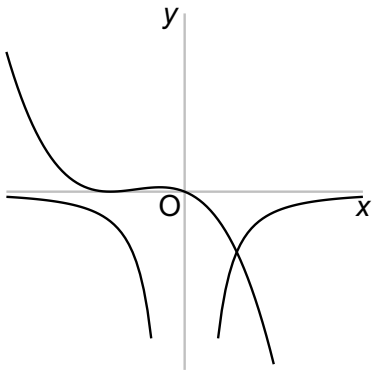
f) $y = -x(x - 1)(x + 1)$, $y = -3x$.

Number of intersections = 3.



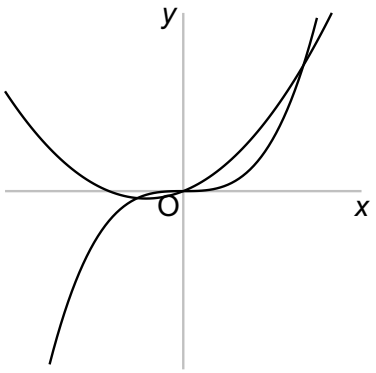
2: a) $y = -\frac{1}{x^2}$, $y = -x(x + 1)^2$.

Number of intersections = 1.



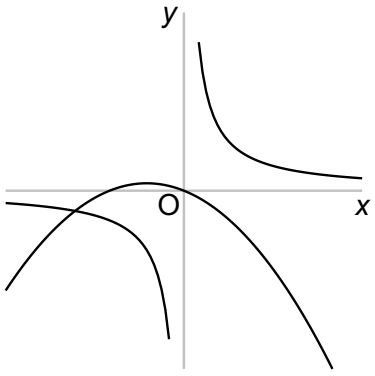
b) $y = x^3$, $y = x(x + 1)$.

Number of intersections = 3.



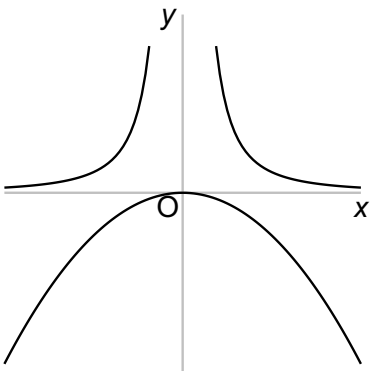
c) $y = \frac{1}{x}$, $y = -x(x + 1)$.

Number of intersections = 1.



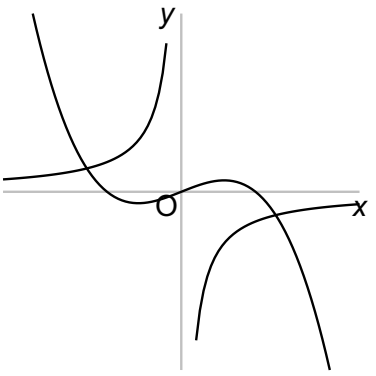
d) $y = \frac{1}{x^2}$, $y = -x^2$.

Number of intersections = 0.



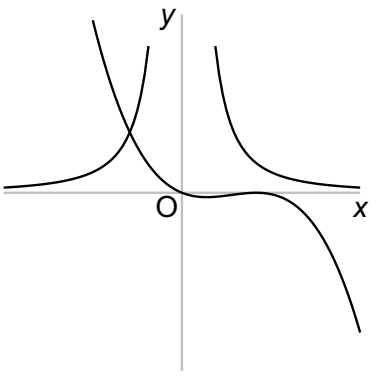
e) $y = -\frac{1}{x}$, $y = -x(x-1)(x+1)$.

Number of intersections = 2.



f) $y = \frac{1}{x^2}$, $y = -x(x-1)^2$.

Number of intersections = 1.



Algebraic Fractions and Proof

MATHSprint

1: Simplify the following as far as possible:

a) $\frac{4z+4}{12z-8}$

b) $\frac{12y-24}{15}$

2: Simplify the following as far as possible:

a) $\frac{w^2+2w}{w^2-3w}$

b) $\frac{12h^2+36h}{16h^2-16h}$

3: Simplify the following as far as possible:

a) $\frac{t^2-4}{t-2}$

b) $\frac{u^2-2u-3}{u^2-u-6}$

4: Simplify the following as far as possible:

a) $\frac{8r^2+41r+5}{6r^2+31r+5}$

b) $\frac{6p^2-p-2}{4p^2-4p-3}$

5: Prove the following statements:

a) Odd + Odd = Odd.

b) The sum of two consecutive even numbers is even.

c) Even + Even = Even.

d) Even × Even = Even.

6: Prove the following statements:

a) The sum of four consecutive numbers is two more than a multiple of 4.

b) The difference between the squares of consecutive odd numbers is a multiple of 8.

c) The difference between the cubes of consecutive odd numbers is two more than a multiple of 24.

d) The difference between the cubes of consecutive numbers is one more than a multiple of 6.

7: Prove the following statements:

a) For all $x \geq 0$ and $y \geq 0$, $x + y \geq \sqrt{x^2 + y^2}$.

b) No three consecutive numbers are all primes.

c) If a and b are both even then ab is a multiple of 4.

d) All square numbers have an odd number of factors.

8: Prove the following statements:

a) For $n > 0$, $4n+2$ is never a square number.

b) For $n > 0$, $n(n-1)(n+1) + n$ is always a cube number.

c) For $n > 2$, n^3-1 is never prime.

d) For $n > 2$, n^2-1 is never prime.

Answers: Algebraic Fractions and Proof

MATHSprint

1: a) $\frac{4(z+1)}{4(3z-2)} = \frac{z+1}{3z-2}$

b) $\frac{3 \times 4(y-2)}{3 \times 5} = \frac{4(y-2)}{5}$

2: a) $\frac{w(w+2)}{w(w-3)} = \frac{w+2}{w-3}$

b) $\frac{4h \times 3(h+3)}{4h \times 4(h-1)} = \frac{3(h+3)}{4(h-1)}$

3: a) $\frac{(t-2)(t+2)}{t-2} = t+2$

b) $\frac{(u-3)(u+1)}{(u-3)(u+2)} = \frac{u+1}{u+2}$

4: a) $\frac{(r+5)(8r+1)}{(r+5)(6r+1)} = \frac{8r+1}{6r+1}$

b) $\frac{(2p+1)(3p-2)}{(2p+1)(2p-3)} = \frac{3p-2}{2p-3}$

5: a) $(2m+1) + (2n+1) = 2[m+n+1]$ ∴Even.

b) $2n + (2n+2) = 2[2n+1]$ ∴Even.

c) $(2m) + (2n) = 2[m+n]$ ∴Even.

d) $(2m)(2n) = 2[2mn]$ ∴Even.

6: a) $(n-1) + (n) + (n+1) + (n+2) = 4n + 2$.

b) $(2n+1)^2 - (2n-1)^2 = 8n$.

c) $(2n+1)^3 - (2n-1)^3 = (8n^3 + 12n^2 + 6n + 1) - (8n^3 - 12n^2 + 6n - 1) = 24[n^2] + 2$.

d) $(n+1)^3 - n^3 = 3n^2 + 3n + 1 = 3[n(n+1)] + 1$
where either n or $n+1$ must be a multiple of 2.

7: a) True: consider $(x+y)^2 = x^2 + 2xy + y^2 \geq x^2 + y^2$
Rooting both sides gives $x + y \geq \sqrt{x^2 + y^2}$.

b) True: At least one of the three numbers must be even; the only even prime is 2 so the list must be 1, 2, 3 - but 1 is not prime.

c) True: $a = 2m$ and $b = 2n$ so $ab = 4[mn]$.

d) True: All factors occur in pairs (f and $\frac{N}{f}$) except for the square root which is duplicated.

8: a) True: $4n+2$ is even and all even square numbers $(2n)^2 = 4n^2$ are divisible by 4 with no remainder; $4n+2$ has a remainder of 2.

b) True: $n(n-1)(n+1) + n = n^3 - n + n = n^3$.

c) True: $n^3-1 = (n-1)(n^2+n+1)$ giving at least 2 factors > 1 when $n > 2$.

d) True: $n^2-1 = (n-1)(n+1)$ giving at least 2 factors > 1 when $n > 2$.

Simultaneous Linear and Quadratic Equations

MATHSprint

Answer on separate paper

1: Solve the following simultaneous equations:

a) $x + y = 3$
$xy = 2$

b) $3x - y = 0$
$xy = 3$

2: Solve the following simultaneous equations:

a) $4x + y = -1$
$2x^2 - xy = 1$

b) $x - 2y = -4$
$3xy + 2y^2 = -4$

3: Solve the following simultaneous equations:

a) $2x - y = -3$
$-2x^2 + 3xy + y^2 = -4$

b) $4x + y = -2$
$-2x^2 + xy + y^2 = 4$

4: Solve the following simultaneous equations:

a) $5x + y = 5$
$xy - 5x = -5$

b) $3x - y = 1$
$xy - 2x - 3y = -6$

5: Solve the following simultaneous equations:

a) $x - 3y = 2$
$x^2 - 2xy - x + 5y = -6$

b) $x + 2y = 4$
$xy + y^2 + 2x + y = 8$

6: Solve the following simultaneous equations:

a) $x - y = -5$
$x^2 + 2xy - y^2 + 4x - 4y = -43$

b) $2x - y = -1$
$-2x^2 - 3xy + y^2 + 3x - 5y = -2$

Answers: Simultaneous Linear and Quadratic Equations

MATHSprint

1: a) $x = (3 - y)$
$xy = 2$
$(3 - y)y = 2$
$-y^2 + 3y - 2 = 0$
$-(y - 2)(y - 1) = 0$

$x = 2, y = 1$
$x = 1, y = 2$

b) $y = (3x)$
$xy = 3$
$x(3x) = 3$
$3x^2 - 3 = 0$
$3(x - 1)(x + 1) = 0$

$x = -1, y = -3$
$x = 1, y = 3$

2: a) $y = (-1 - 4x)$
$2x^2 - xy = 1$
$2x^2 - x(-1 - 4x) = 1$
$6x^2 + x - 1 = 0$
$(3x - 1)(2x + 1) = 0$

$x = -\frac{1}{2}, y = 1$
$x = \frac{1}{3}, y = -\frac{7}{3}$

b) $x = (2y - 4)$
$3xy + 2y^2 = -4$
$3(2y - 4)y + 2y^2 = -4$
$8y^2 - 12y + 4 = 0$
$4(y - 1)(2y - 1) = 0$

$x = -3, y = \frac{1}{2}$
$x = -2, y = 1$

3: a) $y = (2x + 3)$
$-2x^2 + 3xy + y^2 = -4$
$-2x^2 + 3x(2x + 3) + (2x + 3)^2 = -4$
$8x^2 + 21x + 13 = 0$
$(x + 1)(8x + 13) = 0$

$x = -\frac{13}{8}, y = -\frac{1}{4}$
$x = -1, y = 1$

b) $y = (-2 - 4x)$
$-2x^2 + xy + y^2 = 4$
$-2x^2 + x(-2 - 4x) + (-2 - 4x)^2 = 4$
$10x^2 + 14x = 0$
$2x(5x + 7) = 0$

$x = -\frac{7}{5}, y = \frac{18}{5}$
$x = 0, y = -2$

4: a) $y = (5 - 5x)$
$xy - 5x = -5$
$x(5 - 5x) - 5x = -5$
$-5x^2 + 5 = 0$
$-5(x - 1)(x + 1) = 0$

$x = 1, y = 0$
$x = -1, y = 10$

b) $y = (3x - 1)$
$xy - 2x - 3y = -6$
$x(3x - 1) - 2x - 3(3x - 1) = -6$
$3x^2 - 12x + 9 = 0$
$3(x - 3)(x - 1) = 0$

$x = 1, y = 2$
$x = 3, y = 8$

5: a) $x = (3y + 2)$
$x^2 - 2xy - x + 5y = -6$
$(3y + 2)^2 - 2(3y + 2)y - (3y + 2) + 5y = -6$
$3y^2 + 10y + 8 = 0$
$(3y + 4)(y + 2) = 0$

$x = -4, y = -2$
$x = -2, y = -\frac{4}{3}$

b) $x = (4 - 2y)$
$xy + y^2 + 2x + y = 8$
$(4 - 2y)y + y^2 + 2(4 - 2y) + y = 8$
$-y^2 + y = 0$
$-y(y - 1) = 0$

$x = 4, y = 0$
$x = 2, y = 1$

6: a) $x = (y - 5)$

$x^2 + 2xy - y^2 + 4x - 4y = -43$

$(y - 5)^2 + 2(y - 5)y - y^2 + 4(y - 5) - 4y = -43$

$2y^2 - 20y + 48 = 0$

$2(y - 6)(y - 4) = 0$

$x = -1, y = 4$

$x = 1, y = 6$

b) $y = (2x + 1)$

$-2x^2 - 3xy + y^2 + 3x - 5y = -2$

$-2x^2 - 3x(2x + 1) + (2x + 1)^2 + 3x - 5(2x + 1) = -2$

$-4x^2 - 6x - 2 = 0$

$-2(2x + 1)(x + 1) = 0$

$x = -\frac{1}{2}, y = 0$

$x = -1, y = -1$

Name: **Class/Set:**

Box Plots and Outliers

MATHSprint

1: If an outlier is defined as lying more than 1.5 × interquartile range beyond the lower or upper quartile, decide whether each value is an outlier:

a) $Q_1 = 138$, $Q_3 = 174$, value = 233.

b) $Q_1 = 82$, $Q_3 = 154$, value = -16.

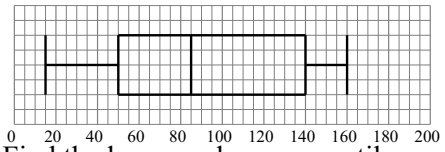
2: If an extreme outlier is defined as lying more than 3 × interquartile range beyond the lower or upper quartile, identify any extreme outliers in the following data sets:

a) -417, -352, 97, 122, 144, 178, 217, 219, 250, 252, 268, 431, 702.

b) 94, 118, 264, 277, 280, 284, 297, 313, 468, 484.

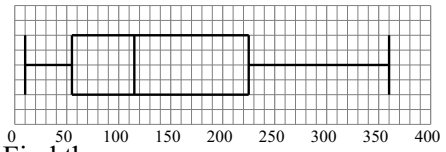
3: Find the required information from the box plot:

a)



Find the lower and upper quartiles.

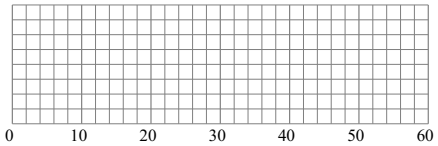
b)



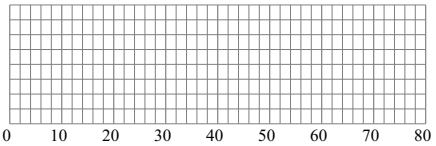
Find the range.

4: Draw the box plot using the given information, given that there is one outlier more than 1.5 × interquartile range beyond the lower or upper quartile:

a) Median = 33, LQ = 29, UQ = 35, min value = 13, max value = 43.

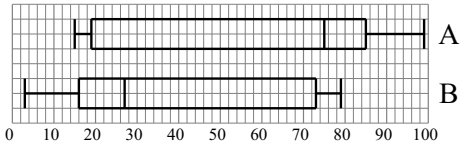


b) Median = 37, LQ = 32, UQ = 40, min value = 29, max value = 66.

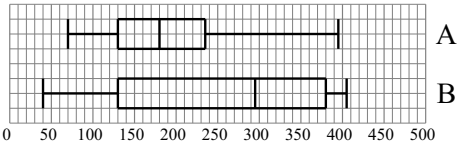


5: For each pair of box plots, give two differences relating to location and spread.

a)



b)



Answers: Box Plots and Outliers

MATHSprint

1: a) Lower boundary = 84 and upper boundary = 228, so 233 **is** an outlier.

b) Lower boundary = −26 and upper boundary = 262, so −16 is **not** an outlier.

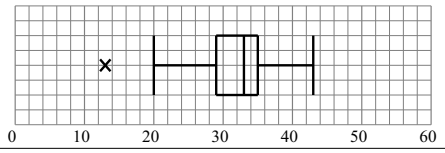
2: a) LQ = 109.5 and UQ = 260 so limits for extreme outliers are −342 and 711.5.
Extreme outliers: −417, −352.

b) LQ = 264 and UQ = 313 so limits for extreme outliers are 117 and 460.
Extreme outliers: 94, 468, 484.

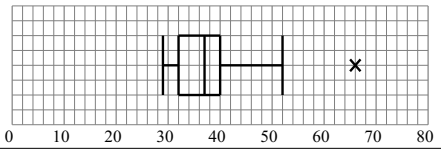
3: a) LQ = 50, UQ = 140

b) Range = 350

4: a)



b)



5: a) A has a higher median/is generally higher than B.
A has a bigger range/overall spread than B.
A has a bigger interquartile range than B.

b) A has a lower median/is generally lower than B.
A has a smaller range/overall spread than B.
A has a smaller interquartile range than B.

Name: **Class/Set:**

Statistics - data collection definitions

MATHSprint

Answer in your files

1: Decide whether the following are examples of qualitative or quantitative data:

a) Favourite food _____

b) Height of student _____

c) Air temperature _____

d) Exam percentage _____

2: Decide whether the following are examples of discrete or continuous data:

a) Number of children _____

b) Time to run 100m _____

c) Capacity of stadium _____

d) Money in bank account _____

3: Answer the following:

a) Give two disadvantages of taking a census.

_____

b) What is a census?

_____

c) What is a sampling unit?

_____

d) Give two advantages of taking a sample.

_____

4: Name the following sampling methods:

a) Start at a random point near the start of the sampling frame; then select items at regularly spaced intervals.

_____

b) Sample the first N people or items that you come across that meet your criteria.

_____

c) Split the sampling frame into groups and take a random sample from each group so that the sample size is proportional to the group size.

_____

d) Each element has an equal chance of being selected; pick them from the sampling frame using non-repeating random numbers.

_____

5: Answer the following:

a) Give two disadvantages of Systematic Sampling.

_____

b) Give two advantages of Stratified Sampling.

_____

c) Give two disadvantages of Stratified Sampling.

_____

d) Give two advantages of Systematic Sampling.

_____

Answers: Statistics - data collection definitions

MATHSprint

1: a) QUALitative. b) QUANTitative. c) QUANTitative. d) QUANTitative.

2: a) Discrete. b) Continuous. c) Discrete. d) Discrete.

3: a) Census: expensive - takes a long time - can't be used if the test is destructive

b) A census observes or measures every single member of a population

c) A sampling unit is an individual item or person in the population

d) Sample: cheaper - quicker - more questions can be asked - okay if the test is destructive

4: a) Systematic Sampling b) Opportunity (or Convenience) Sampling

c) Stratified Sampling d) Simple Random Sampling

5: a) Systematic: errors if there is an underlying periodic pattern - a sampling frame is needed

b) Stratified: reflects the population more accurately - subgroups are fairly represented - can compare different subgroups

c) Stratified: sampling frame must be sorted into strata - clustering can still happen within each stratum

d) Systematic: easy & cheap - avoids picking clusters - suitable for large populations - only one random number needed

Name: **Class/Set:**

Mean, Variance & SD calculations

MATHSprint

1: Calculate the following (round to 4 decimal places if necessary):

a) If $\bar{x} = 67$, variance $\sigma^2 = 58$ and $\Sigma fx^2 = 272820$, find Σf.

_____

b) If variance $\sigma^2 = 41$, $\Sigma fx = 4680$ and $\Sigma fx^2 = 307152$, find Σf.

_____

c) If $\Sigma f = 50$, variance $\sigma^2 = 62$ and $\Sigma fx^2 = 171300$, find the mean $\bar{x}$.

_____

d) If $\Sigma f = 27$, $\Sigma fx = 729$ and $\Sigma fx^2 = 21654$, find the variance σ^2.

_____

2: Calculate the following (round to 4 decimal places if necessary):

a) If variance $\sigma^2 = 88$, $\Sigma x = 3240$ and $\Sigma x^2 = 294768$, find n.

_____

b) If $n = 16$, $\Sigma x = 208$ and $\Sigma x^2 = 2880$, find the standard deviation σ.

_____

c) If $n = 54$, variance $\sigma^2 = 87$ and $\Sigma x^2 = 8154$, find the mean $\bar{x}$.

_____

d) If $\bar{x} = 45$, variance $\sigma^2 = 34$ and $\Sigma x^2 = 61770$, find n.

_____

3: Calculate the following (round to 4 decimal places if necessary):

a) If a data set has $n = 14$, mean = 54 and variance = 7, find what value must be added to change the mean to 64.

_____

b) If a data set has $n = 7$, $\Sigma x = 651$ and $\Sigma x^2 = 60767$, find the standard deviation when an extra value 102 is added.

_____

c) If a data set has $n = 39$, $\Sigma x = 2808$ and $\Sigma x^2 = 204867$, find the variance when an extra value 74 is added.

_____

d) If a data set has $n = 4$, mean = 61 and standard deviation = $\sqrt{23}$, find the standard deviation when an extra value 59 is added.

_____

4: Calculate the following (round to 4 decimal places if necessary):

a) If one data set has $n = 45$, mean = 96 and variance = 2 and another has $n = 64$, mean = 78 and variance = 21, find the variance of the combined data set.

_____

b) If one data set has $n = 20$, $\Sigma x = 200$ and $\Sigma x^2 = 2760$ and another has $n = 18$, $\Sigma x = 1476$ and $\Sigma x^2 = 122652$, find the standard deviation of the combined data set.

_____

c) If one data set has $n = 9$, mean = 68 and variance = 27 and another has $n = 24$, mean = 46 and variance = 1, find the mean of the combined data set.

_____

d) If one data set has $n = 25$, $\Sigma x = 550$ and $\Sigma x^2 = 13025$ and another has $n = 48$, $\Sigma x = 144$ and $\Sigma x^2 = 624$, find the variance of the combined data set.

_____

Answers: Mean, Variance & SD calculations

MATHSprint

1: a) $\Sigma f = \frac{272820}{58 + 67^2} = 60$

b) $\Sigma f = 72$

c) Mean $\bar{x} = \sqrt{\frac{171300}{50} - 62} = 58$

d) Variance $\sigma^2 = \frac{21654}{27} - \left(\frac{729}{27}\right)^2 = 73$

2: a) $n = 36$

b) SD $\sigma = \sqrt{\frac{2880}{16} - \left(\frac{208}{16}\right)^2} = \sqrt{11} = 3.3166$ [4 dp].

c) Mean $\bar{x} = \sqrt{\frac{8154}{54} - 87} = 8$

d) $n = \frac{61770}{34 + 45^2} = 30$

3: a) Value $= 15 \times 64 - 54 \times 14 = 204$

b) SD $= \sqrt{\frac{60767 + 102^2}{8} - \left(\frac{651 + 102}{8}\right)^2} = 6.0712$

c) Variance $= \frac{204867 + 74^2}{40} - \left(\frac{2808 + 74}{40}\right)^2 = 67.3725$

d) SD $= \sqrt{\frac{(23 + 61^2) \times 4 + 59^2}{5} - \left(\frac{61 \times 4 + 59}{5}\right)^2} = 4.3635$

4: a) Variance $= \frac{(2 + 96^2) \times 45 + (21 + 78^2) \times 64}{45 + 64} - \left(\frac{96 \times 45 + 78 \times 64}{45 + 64}\right)^2 = 91.6948$

b) SD $= \sqrt{\frac{2760 + 122652}{20 + 18} - \left(\frac{200 + 1476}{20 + 18}\right)^2} = 36.8109$

c) Mean $= \frac{68 \times 9 + 46 \times 24}{9 + 24} = 52.0000$

d) Variance $= \frac{13025 + 624}{25 + 48} - \left(\frac{550 + 144}{25 + 48}\right)^2 = 96.5924$

Name: **Class/Set:**

Normal Distribution

MATHSprint

1: Calculate the following (round to 4 decimal places if necessary):

a) The random variable $X \sim N(\mu, 10^2)$ and $P(X \leq -7) = 0.0808$. Find the value of μ.

_____

b) The random variable $X \sim N(3, \sigma^2)$ and $P(X > 2) = 0.5362$. Find the value of σ.

_____

2: Calculate the following (round to 4 decimal places if necessary):

a) The time taken to brush one's teeth is modelled by a normal distribution with a mean of μ seconds and a standard deviation of 30 seconds. If a random person is selected, the probability of the time being greater than 122.3 seconds is 0.1903. Find the value of μ.

_____

b) The length of a mature blue whale is modelled by a normal distribution with a mean of 23 metres and a standard deviation of σ metres. If a random blue whale is selected, the probability of the length being less than 15.2 metres is 0.0256. Find the value of σ.

_____

3: Calculate the following (round to 4 decimal places if necessary):

a) The random variable $X \sim N(\mu, \sigma^2)$. If $P(X < -5) = 0.0256$ and $P(X < 60) = 0.9032$, find the values of μ and σ.

_____

b) The random variable $X \sim N(\mu, \sigma^2)$. If $P(X \leq 81) = 0.7475$ and $P(X \geq 91) = 0.1108$, find the values of μ and σ.

_____

4: Calculate the following (round to 4 decimal places if necessary):

a) The mass of a lettuce is modelled by a normal distribution with a mean of μ grams and a standard deviation of σ grams. If a random lettuce is selected, the probability of the mass being less than 476.4 grams is 0.1177 and the probability of it being less than 477 grams is 0.1217. Find the values of μ and σ.

_____

b) The mass of a bag of sweets is modelled by a normal distribution with a mean of μ grams and a standard deviation of σ grams. If a random bag of sweets is selected, the probability of the mass being less than 108 grams is 0.9772 and the probability of it being greater than 107.8 grams is 0.0287. Find the values of μ and σ.

_____

Answers: Normal Distribution

MATHSprint

1: a) $\frac{-7-\mu}{10} = -1.400$ so $\mu = 7$. b) $\frac{2-3}{\sigma} = -0.091$ so $\sigma = 11$.

2: a) $\frac{122.3-\mu}{30} = 0.877$ so $\mu = 96$ seconds. b) $\frac{15.2-23}{\sigma} = -1.950$ so $\sigma = 4$ metres.

3: a) $\frac{-5-\mu}{\sigma} = -1.950$ and $\frac{60-\mu}{\sigma} = 1.300$ so $\mu = 34$ and $\sigma = 20$.

b) $\frac{81-\mu}{\sigma} = 0.667$ and $\frac{91-\mu}{\sigma} = 1.222$ so $\mu = 69$ and $\sigma = 18$.

4: a) $\frac{476.4-\mu}{\sigma} = -1.187$ and $\frac{477-\mu}{\sigma} = -1.167$ so $\mu = 512$ grams and $\sigma = 30$ grams.

b) $\frac{108-\mu}{\sigma} = 2.000$ and $\frac{107.8-\mu}{\sigma} = 1.900$ so $\mu = 104$ grams and $\sigma = 2$ grams.

Name: **Class/Set:**

PMCC Hypothesis Testing

MATHSprint

1: Work out the following, giving any numbers correct to 4 decimal places if necessary:

a) Wilf thinks that there is a negative correlation between the amount of exercise per week and body mass index and takes a sample of 5 data pairs, getting a PMCC of -0.7905.
Testing Wilf's claim at the 2.5% level, is this PMCC value in the critical region?

_____

b) Vijay reckons that there is a correlation between the cost of a car and the reliability of a car and takes a sample of 16 data pairs, getting a PMCC of -0.4706.
Testing Vijay's claim at the 20% level, is this PMCC value in the critical region?

_____

2: Work out the following, giving any numbers correct to 4 decimal places if necessary:

a) Sophie is pretty sure that there is a positive correlation between air pressure and temperature and takes a sample of 7 data pairs, getting a PMCC of 0.7496.
Carry out a hypothesis test at the 1% significance level to investigate Sophie's claim.

_____

b) Georgia conjectures that there is a correlation between hand span and number of T-shirts owned and takes a sample of 10 data pairs, getting a PMCC of 0.8117.
Carry out a hypothesis test at the 1% significance level to investigate Georgia's claim.

_____

Answers: PMCC Hypothesis Testing

MATHSprint

1: a) 1-tailed test at 2.5% level, $n = 5$: critical region is $r \leq -0.8783$,
so -0.7905 is **not** in the critical region.

b) 2-tailed test at 20% level, $n = 16$: critical region is $r \leq -0.3383$ or $r \geq 0.3383$,
so -0.4706 is in the critical region.

2: a) H_0: $\rho = 0$ and H_1: $\rho \geq 0$.
1-tailed test at 1% level, $n = 7$: critical region is $r \geq 0.8329$,
so 0.7496 is **not** in the critical region.
There is insufficient evidence at the 1% level to reject H_0, the hypothesis that air pressure and temperature are not correlated.

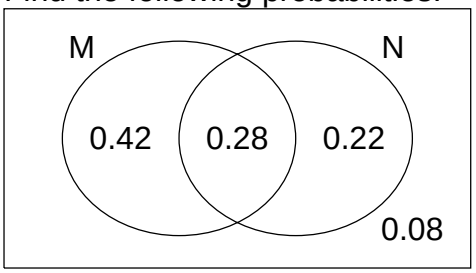
b) H_0: $\rho = 0$ and H_1: $\rho \neq 0$.
2-tailed test at 1% level, $n = 10$: critical region is $r \leq -0.7646$ or $r \geq 0.7646$,
so 0.8117 is in the critical region.
There is evidence at the 1% level to reject H_0, supporting the alternative hypothesis that there is a correlation between hand span and number of T-shirts owned.

Name: Class/Set:

Venn Diagrams

MATHSprint

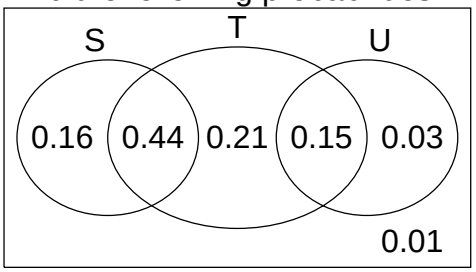
1: Find the following probabilities.



a) P(N ¢) b) P(M Ç N) c) P(M) d) P(N)

_____ _____ _____ _____

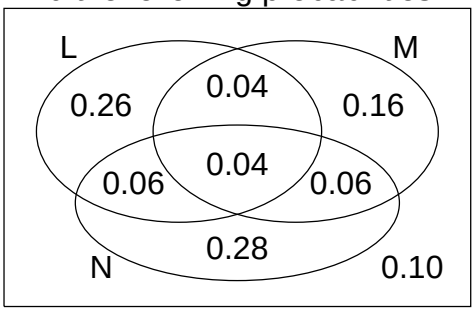
2: Find the following probabilities.



a) P(T Ç U) b) P(S Ç U) c) P(S Ç T) d) P(T ¢)

_____ _____ _____ _____

3: Find the following probabilities.



a) P(M È N) b) P((L È M È N) ¢) c) P((L È N) ¢)

_____ _____ _____

d) P(N Ç M ¢) e) P((L Ç M ¢) È (L ¢ Ç M)) f) P(L È M È N)

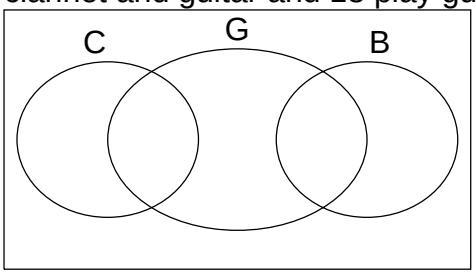
_____ _____ _____

g) P((M È N) ¢) h) P((L È M) ¢)

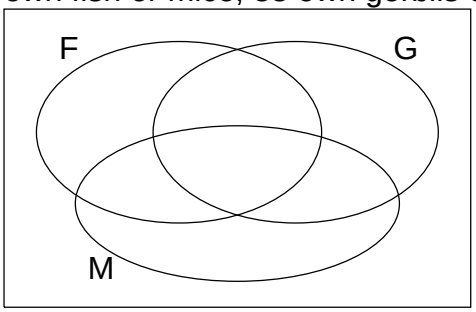
_____ _____

4: Complete the Venn diagram using the information provided.

a) A survey of 51 musicians finds that 17 play clarinet, 14 play guitar, 8 play bassoon, 9 play both clarinet and guitar and 18 play guitar or bassoon.

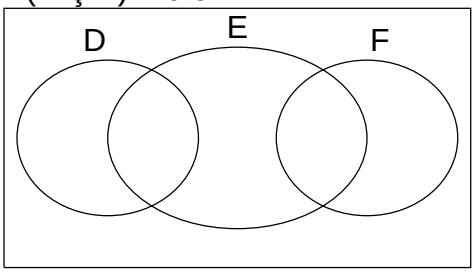


b) Out of 51 children, 27 own fish, 18 own gerbils, 28 own mice, 6 own both fish and gerbils, 41 own fish or mice, 38 own gerbils or mice and 1 own fish, gerbils and mice.



5: Complete the Venn diagram using the information provided.

a) For events D, E and F, P(D) = 0.20, P(E) = 0.10, P(F) = 0.26, P(D Ç E) = 0.07 and P(E Ç F) = 0.02.



b) For events T and U, P(T ¢) = 0.10, P(U ¢) = 0.40 and P(T Ç U) = 0.53.